Investigation of a Perpetual Kinetic Pendulum

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In Search of a System:

The kinetic pendulum is a system submitted to the Visual Education Project's Perpetual Motion Competition. It consists of a rigid pendulum upon a semicircular roller. In the absence of friction, there theoretically exists a regime where the pendulum never tips, but eternally oscillates in stable harmonic motion. We set out to investigate: "What parameters provide the longest elapsed time from start till when the pendulum topples?"

From Kinetic Pendulum Demonstration Video

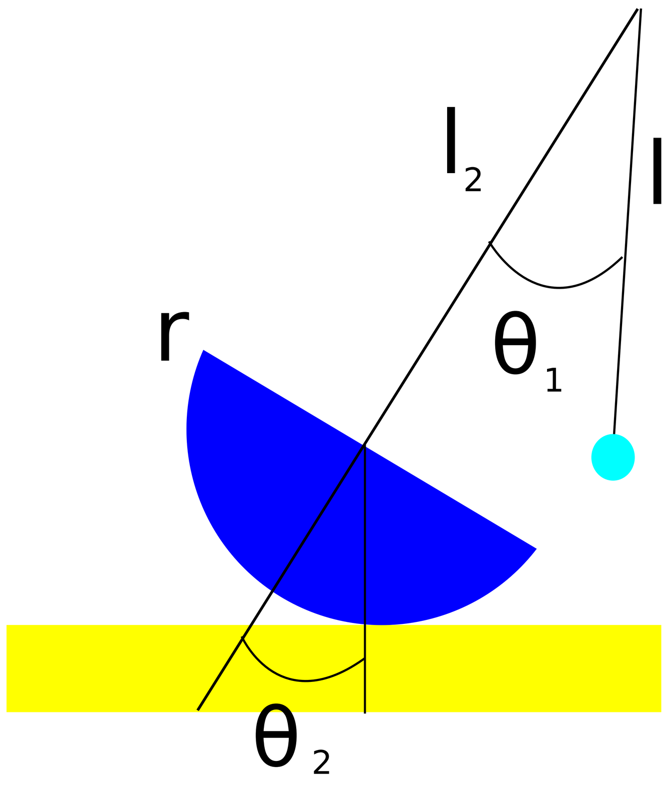
Modelling Approach:

Cartesian

$$X_b = \ell_3 sin(\theta_2) + \ell_1 sin(\theta_1 - \theta_2) + r\theta_2$$
$$Y_b = \ell_3 cos(\theta_2) - \ell_1 cos(\theta_1 - \theta_2) + r$$
$$\dot{X}_b = \ell_3 cos(\theta_2)\dot{\theta}_2 + \ell_1 cos(\theta_1 - \theta_2)(\dot{\theta}_1 - \dot{\theta}_2) + r\dot{\theta}_2$$
$$\dot{Y}_b = \ell_3 sin(\theta_2)\dot{\theta}_2 + \ell_1 sin(\theta_1 - \theta_2)(\dot{\theta}_1 - \dot{\theta}_2)$$

Energy

$$PE_{ball} = m_b g\Big(r + \ell_3 cos(\theta_2) - \ell_1 cos(\theta_1 - \theta_2)\Big)$$
$$PE_{roller} = m_r g\Big(r - cos(\theta_2)\frac{4r}{3\pi}\Big)$$
$$KE_{ball} = \frac{1}{2} m_b\Big(\big(\dot{X}_b\big)^2 + \big(\dot{Y}_b\big)^2\Big)$$
$$KE_{roller} = \frac{1}{2} m_r \dot{\theta}_2^{\,2}\Big(\big(r - \frac{4r}{3\pi}cos(\theta_2)\big)^2 + \big(sin(\theta_2)\big)^2\Big)$$



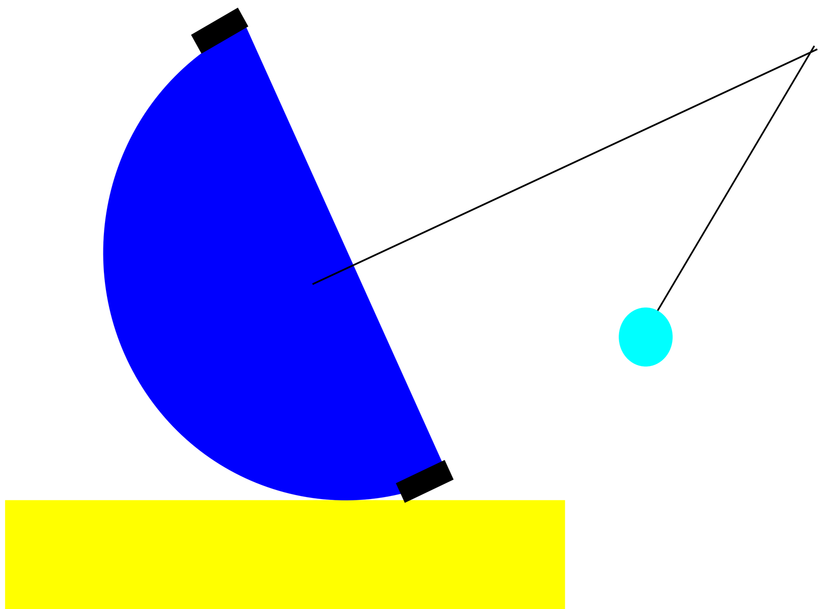
System Diagram

ABSTRACT

Inspired by a proposed perpetual motion machine, we investigate the optimal system parameters for a kinetic pendulum to achieve stable harmonic motion, employing a two point mass lagrangian mechanics model. Using this simulation we determine that the time till the pendulum topples is dependent upon the ratio between roller mass and ball mass, not magnitude. We also determine that the elapsed time till the pendulum topples is directly proportional to the magnitude of the frame and arm lengths, and dependent upon their ratio. We find that smaller mass ratios of ball to roller yield a stabler system, however, there exists an optimal length ratio of approximately 3:4. We also find that there exists a narrow optimal band of initial angle pairs, where the main stable regime parameters are linearly dependent.

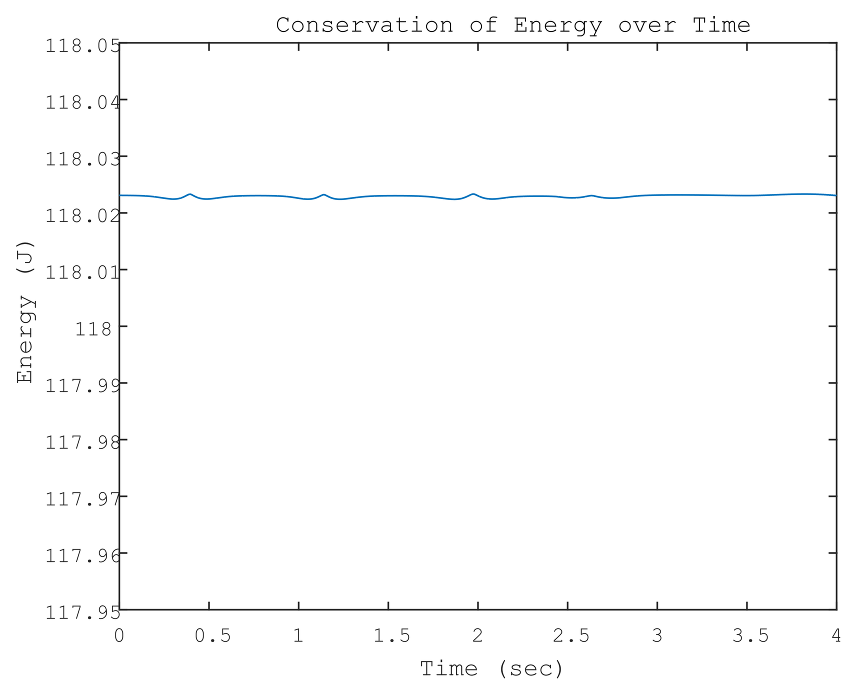
Choices and Limitations:

Primary: Frame and Pendulum arm not included in model. Additionally, the roller is modelled as a point mass, which does nto account for more acccurate complex dynamics.
Secondary: Friction and damping are not included in the system. In the demonstration video, bumpers are used to restrain the roller from toppling. This creates a much stabler system, which we chose not to include.



Bumpers Stopping the Roller from Toppling

Qualitative and Quantitative Verification:

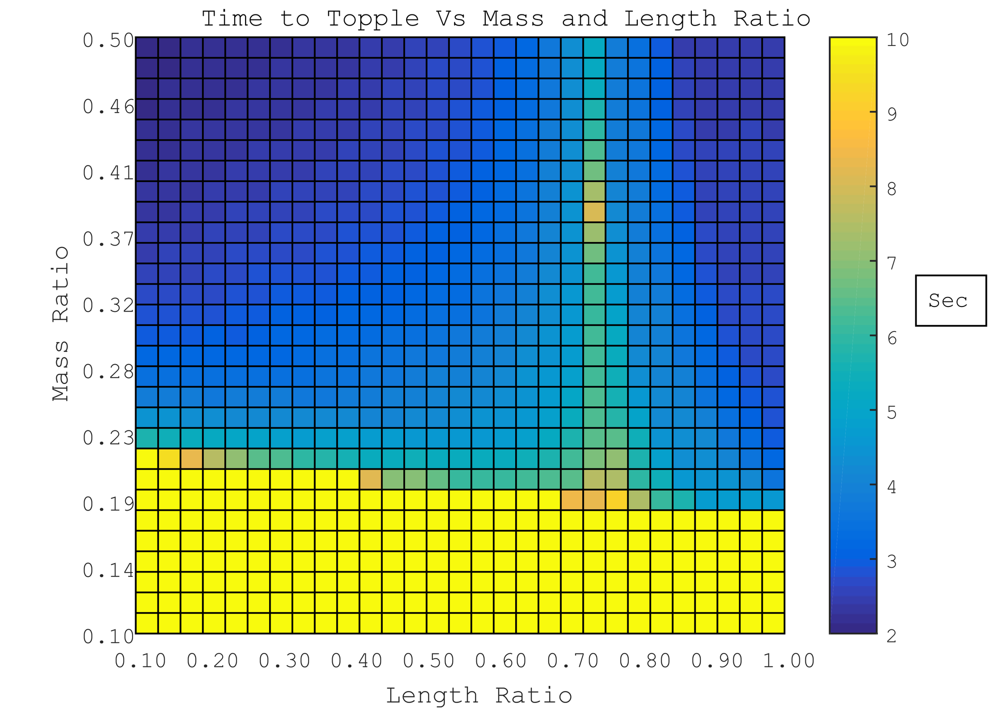


Energy is Conserved, supporting the validity of our model implementation

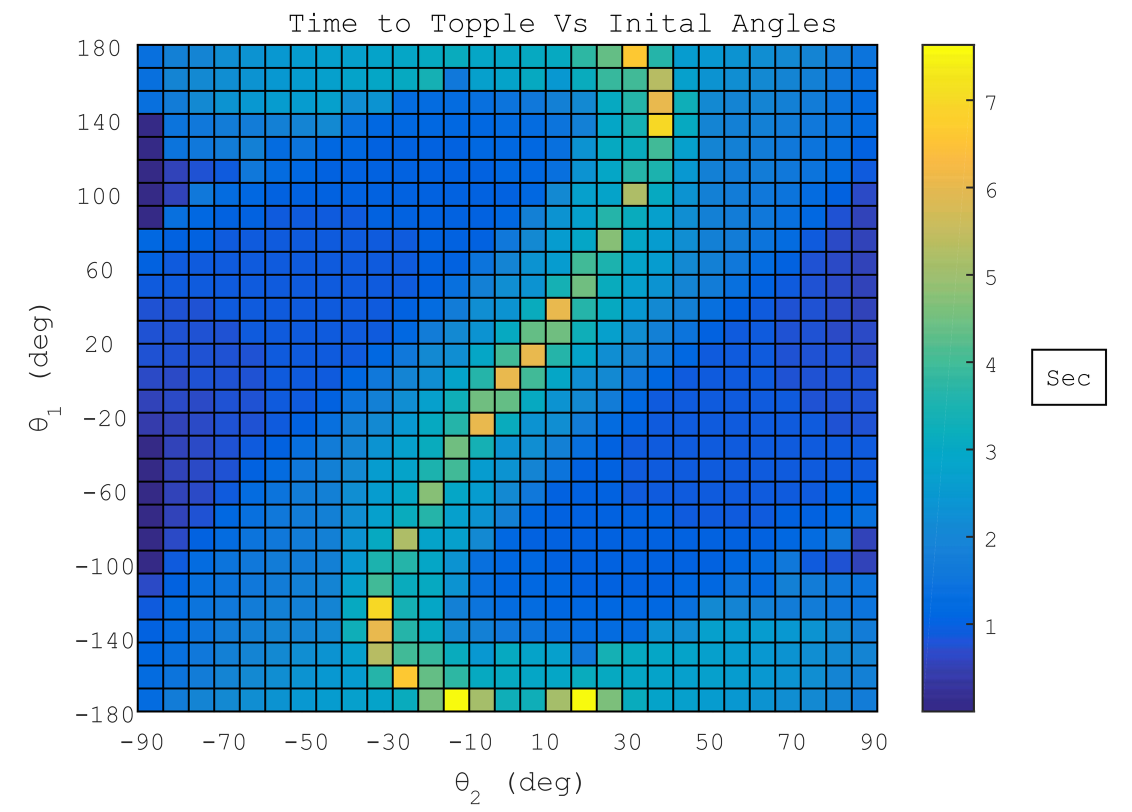
Qualitatively, animations of our model appear consistent with expected behavior. Quanititatively, total energy is conserved within our model. There are some small-drifts, but these are a product of the discrete ODE solver operating on a continuous system. The magnitude of these drifts is proportional to the tolerance of our solver, demonstrating its independence from our model.

Notable Regimes:

Taking fixed intial angles, we analyzed the effects of mass and length ratios. A smaller relative mass of ball creates a system with less initial energy: generally stabler. Regarding length ratio, a localized trend was observed yielding a much stabler system. With these parameters, initial angle conditions were optimized. The combination of these factors provides a system with notably stable behavior relative to initial energy.



Results show an optimal length ratio of ~3:4



Optimal angle conditions are linearly dependent between -90 and 90 degrees, with emergent behavior outside of this range

Further Possibilities:

In further iterations, a more complex model that accounts for the frame and pendulum arm could yield more realistic results. This can also be said for a model that includes friction. With the current model, relationships between the radius of the roller and the other parameters could be explored and optimized.