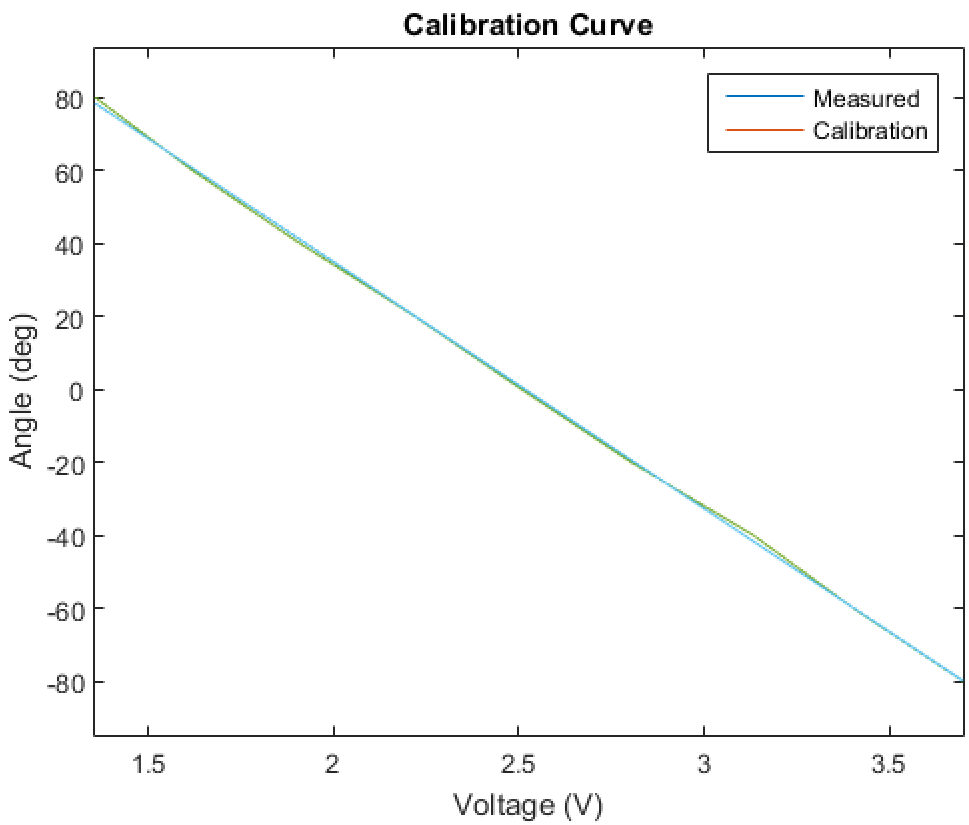
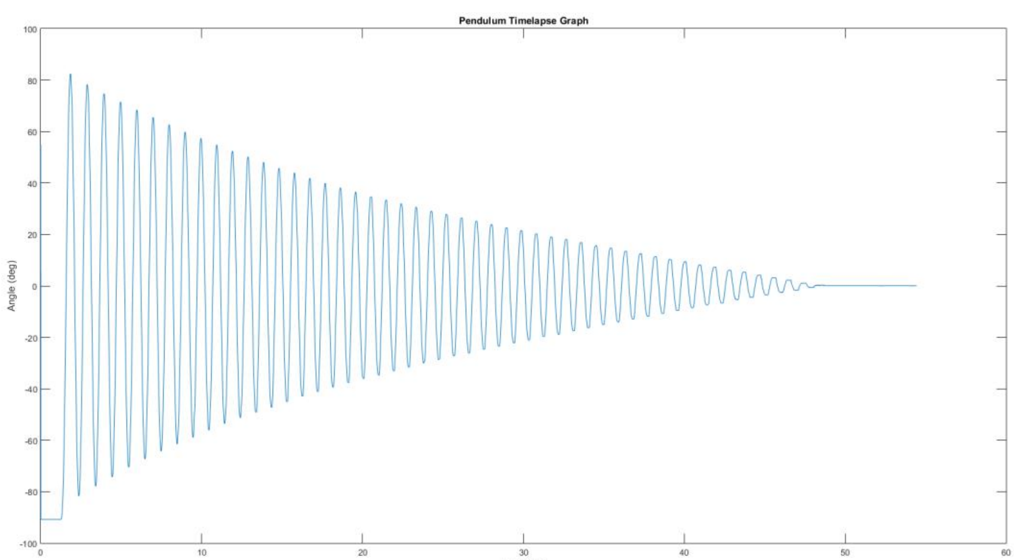
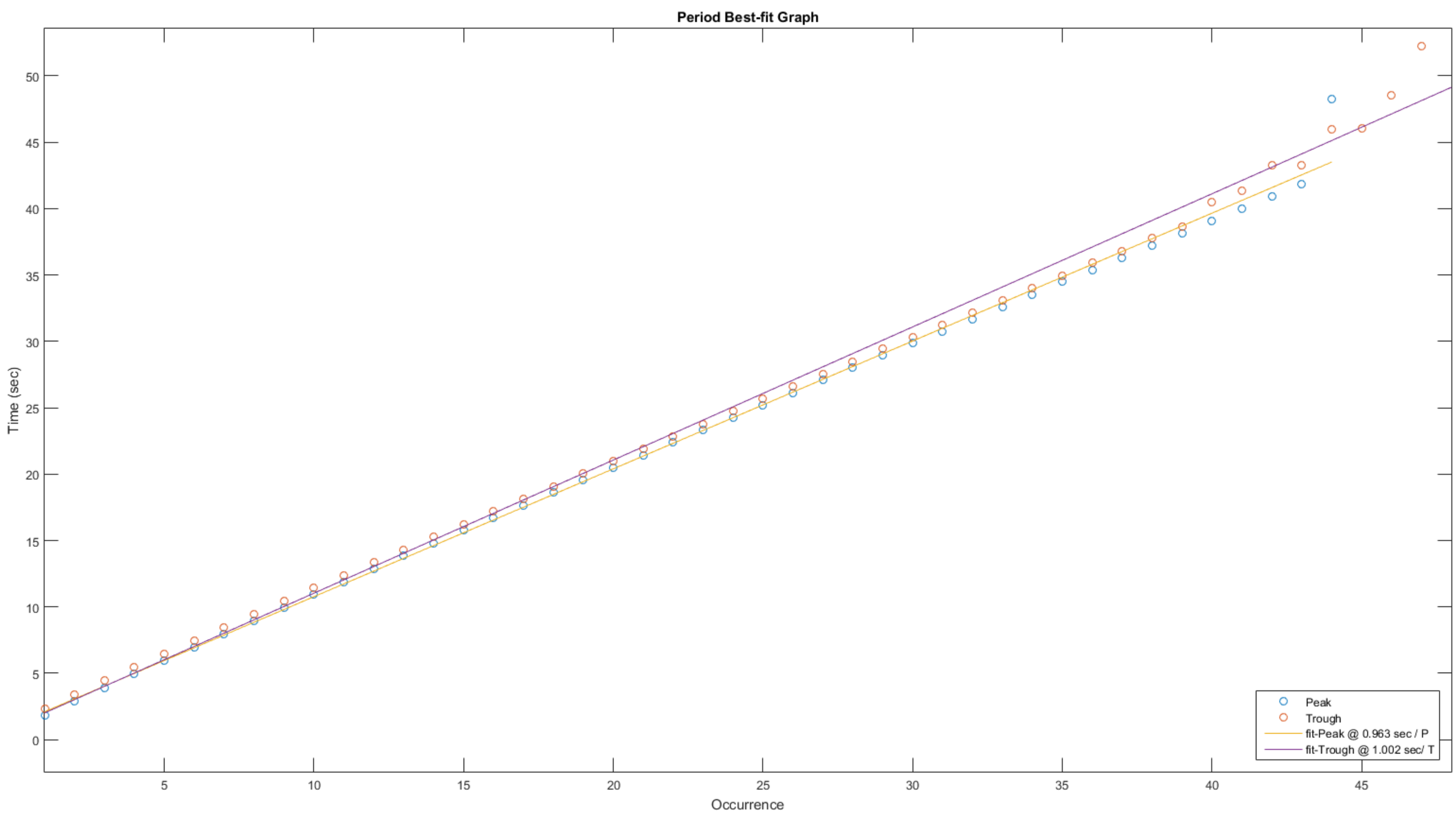
ISIM Lab #1 - Pendulum

Jamie Cho

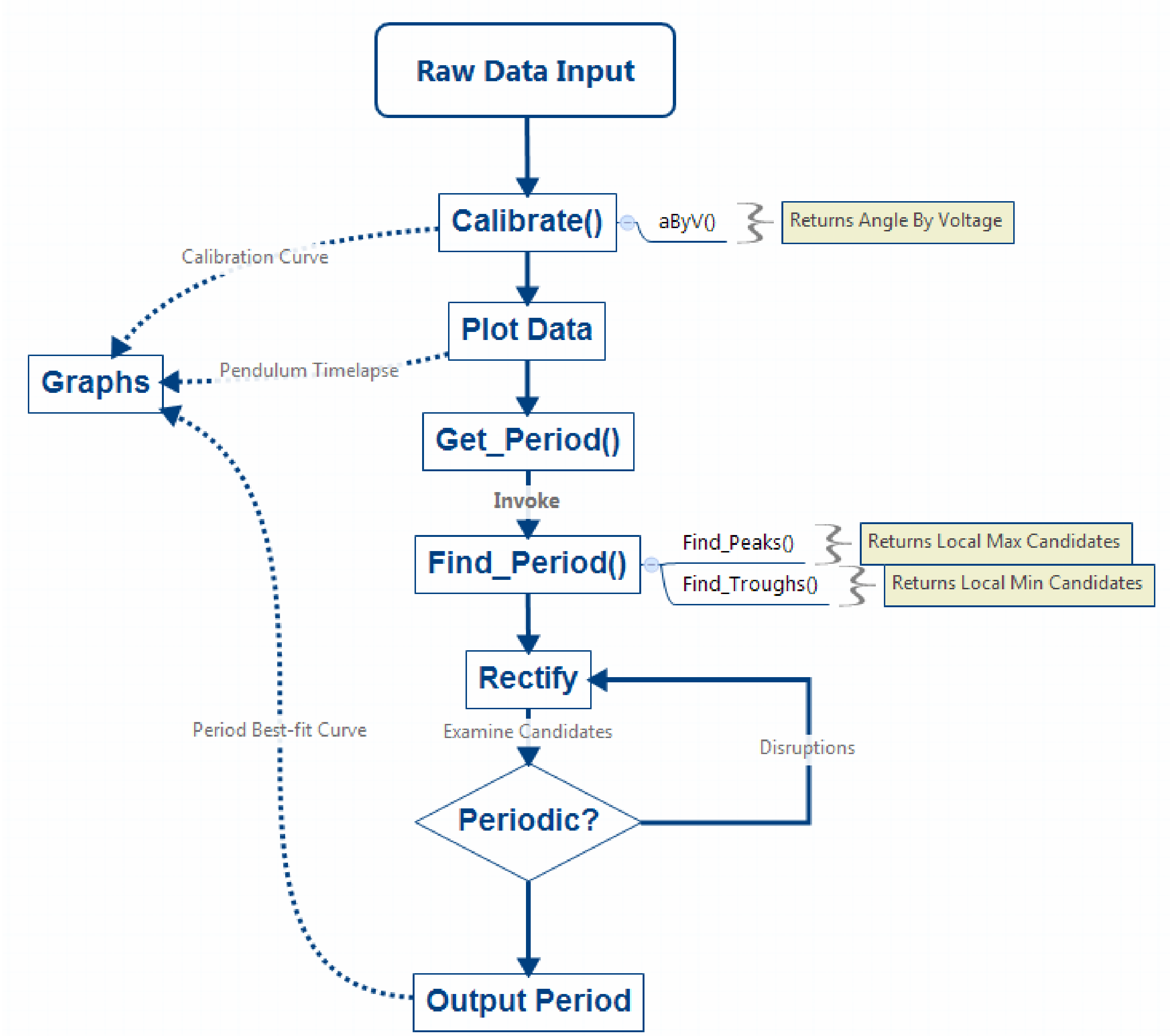
1. Figures






Period Best-fit Graph
Time (sec)
Occurrence
0
5
10
15
20
25
30
35
40
45
50
5
10
15
20
25
30
35
40
45
Peak
Trough
fit-Peak @ 0.963 sec / P
fit-Trough @ 1.002 sec/ T

2. Flowchart



3. MATLAB Command Line & Period Value

```
>> Calibration()
>>  plot(Times+30,aByV(C1V)); %Pendulum Timelapse Graph
>> get_Period()
Period is 0.963 seconds(Peaks), 1.002 seconds(Troughs)
 Ave. to 0.982 seconds
```

4. MATLAB Scripts

Calibration.m

```
%precondition : Cal_Voltage & Cal_Angle are imported Data

c_fit = polyfit(Cal_Voltage, Cal_Angle, 1);
plot(Cal_Voltage, Cal_Angle);
```

```
 hold on
aByV = @(x)x*c_fit(1) + c_fit(2); %Angle By Voltage
ezplot(aByV, Cal_Voltage);
legend('Measured','Calibration');
title('Calibration Curve')
xlabel('Voltage (V)');
ylabel('Angle (deg)');
```

get_Period.m

```
%precondition : C1V is the raw Voltage, Times range in -30:30
%p = Peak, t = Trough
 [p,t] = find_Period(aByV(C1V),Times+30);

flag = true;

while(flag) %rectification feedback loop
    flag = false;
    scale_p = [1:length(p)];
    scale_t = [1:length(t)];

    p_fit = polyfit(scale_p,p,1);
    t_fit = polyfit(scale_t,t,1);

    i = 2;
    while(i < length(p))
        if( p(i+1) - p(i-1) < p_fit(1)*5/4) %in discord with other data
            p(i) = [];
            flag = true; %indicates change
        else
            i = i+1;
        end
    end

    i = 2; %repeat same process with troughs
    while(i < length(t))
        if( t(i+1) - t(i-1) < t_fit(1)*5/4)
           t(i) = [];
           flag = true;
        else
            i = i+1;
        end
    end

end %ends when nothing is out of place


plot(scale_p,p,'o');
hold on
plot(scale_t,t,'o'); %raw periods

fit_func_p = @(x) x*p_fit(1) + p_fit(2);
ezplot(fit_func_p,scale_p);

fit_func_t = @(x) x*t_fit(1) + t_fit(2);
```

```
ezplot(fit_func_t,scale_t); %polynomials from period data

p_label = sprintf('fit-Peak @ %.3f sec / P', p_fit(1));
t_label = sprintf('fit-Trough @ %.3f sec/ T', t_fit(1));

l = legend('Peak','Trough',p_label,t_label, 'Location','SouthEast');
title('Period Best-fit Graph');
xlabel('Occurrence');
ylabel('Time (sec)');

msg = sprintf('period is %.3f seconds(Peaks), %.3f seconds(Troughs)\n Ave.
to %.3f seconds', p_fit(1), t_fit(1), (p_fit(1)+t_fit(1))/2);
disp(msg);
```

find_Period.m

```
function [p,t] = find_Period(dat,Time)
p = find_Peaks(dat,Time);
t = find_Troughs(dat,Time);
end

function ret = find_Peaks(dat, time)
r = 20; %range +-. at the sampling rate of 60/7260 (~0.008) seconds/sample,
this amounts to +- 0.165 seconds.
ret = [];
len = length(dat);
prevval = 0;
    for val = 1+r:len-r
        if(abs(prevval -dat(val)) < 0.2) %ignore plateaus at top, leeway of
0.2 degrees difference.
            continue;
        else
            prevval = dat(val);
        end
        if(isLocalMax(dat(val), dat(val-r:val+r))) % = if Peak
            ret(end+1) = time(val); %append new Peak
            prevval = dat(val);
        else
            %disp(val);
        end
    end
end

function ret = find_Troughs(dat, time) %ditto
r = 20;
ret = [];
len = length(dat);
prevval = 0;
    for val = 1+r:len-r
        if(abs(prevval -dat(val)) < 0.2)
            continue;
        else
            prevval = dat(val);
        end
        if(isLocalMin(dat(val), dat(val-r:val+r))) % = if Trough
            ret(end+1) = time(val); %append new Trough
```

```
            prevval = dat(val);
        else
            %disp(val);
        end
    end
end


function ret = isLocalMax(val, arr)
    ret = true;
    for dat = arr'
       if(dat > val)
           ret = false;
           break;
       end
    end

    if(arr(1) == val || arr(end) == val)
        ret = false;
    end

end
function ret = isLocalMin(val, arr)
    ret = true;
    for dat = arr'
       if(dat < val)
           ret = false;
           break;
       end
    end

    if(arr(1) == val || arr(end) == val)
        ret = false;
    end

end
```

ISIM Lab 2 – Strain Gauge

Jamie Cho

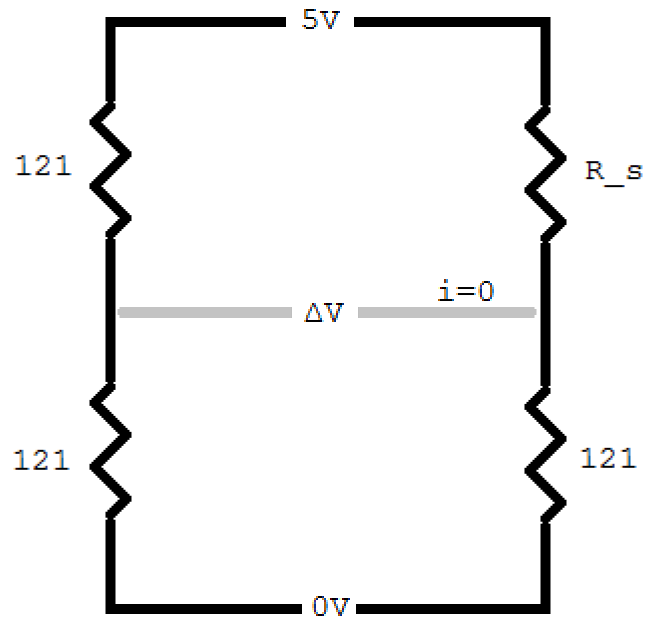


Figure 1. The diagram of the Strain Gauge circuit, which will be referred to extensively in this document. For simplicity of representation, potentiometers and the strain gauge setup were removed; more specific diagram is already detailed in the lab handout. R_s is R_{strain}.

1)

The equation derived from the above circuit is:

$$\Delta V_{amp} = \left(1 + \frac{100000}{R_{gain}}\right) * \left(\left(5 - \frac{5}{121 + 121} * 121\right) - \left(5 - \frac{5 * R_{strain}}{121 + R_{strain}}\right)\right)$$

Notice that for the parallel resistors, R_{strain} ought to be calculated beforehand.

Equivalently, solving this for $R_{strain,}$:

$$R_{strain} = -\frac{121 * (R_{gain} * (2 * V + 5) + 500000)}{R_{gain} * (2 * V - 5) - 500000}$$

The following MATLAB script is the implementation of the first equation. It also reveals the derivation of the first formula step-by-step:

```
function V = V_amp(R_strain, R_gain)
% Independent Variables

Gain = 1 + 100000/R_gain;
I_left = 5/(121 + 121);
I_right = 5/(121+ R_strain);
```

```
V_left = 5 - I_left*121;
V_right = 5 - I_right * R_strain;
V_raw = V_left - V_right;
V = V_raw * Gain;

end
```

Running the following command line:

```
R_eq = @(R1,R2) 1/(1/R1 + 1/R2);

R_strain = R_eq(121,100*1000);

R_gain = 4990;

disp (V_amp(R_strain,R_gain));
```

displays -0.0318, which is consistent with measured value (-36 mV = -0.36V ≈ -.0318 V)

The following chart depicts the data obtained from repeated iterations:

Resistance(Ω)	Theoretical (V)	Experimental (V)	Discrepancy (%)
100 K	-0.0318	-.036	-13.2%
499 K	-0.0064	-.008	-25%
1 M	-0.0032	-.004	-25%

To account for the (relatively big) discrepancy, it is important to note that – as indicated above – I used a 4.99KΩ resistor for the amplifier, which undermined the resolution of the measure; 200Ω resistor consistently failed to perform as anticipated on my circuit, outputting in the range of 2.36V regardless of change in R_{strain}. Baffled by the results, I consulted my peers, one of whom – Eric Miller – attributed this phenomenon to the limitation in the Analog Discovery's capacity of measurement. The input voltage was simply too big to display a meaningful value. At his advice, I changed the resistor from 200Ω resistor to 4.99KΩ resistor, after which the experimental data was coherent with the theoretical data.

That is, though the discrepancy may seem large, it is significantly similar to the theoretical value, especially considering that even a delicate change – in this experiment – brings about great ramifications. After the extensive verification, the results may now be applied in the following section.

2)

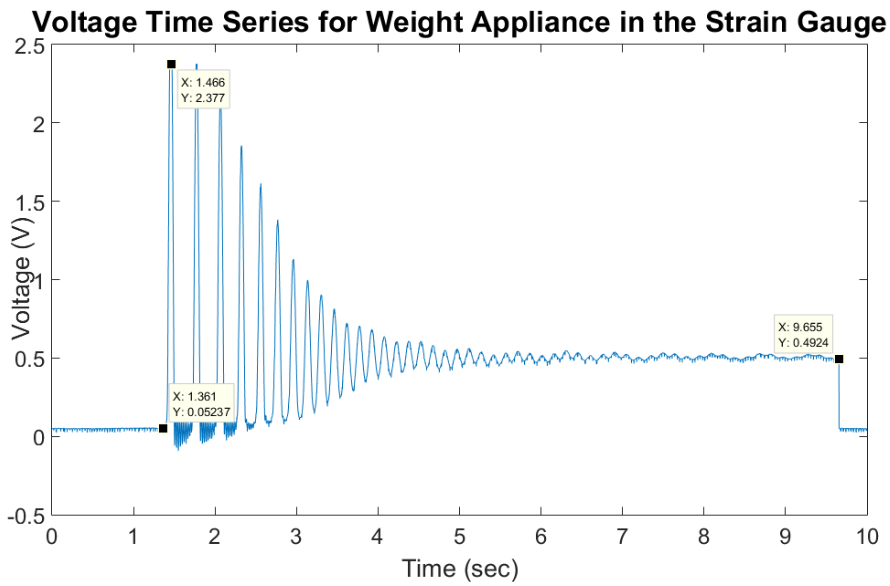


Figure 2. Amplified Voltage Time Series; it is unclear from the figure, but the initial jump was from 0.05237 to 2.377 Volts, later settling at 0.4924 Volts.

In order to measure the minute change in the strain gauge, the 4.99KΩ resistor was supplanted with 200Ω resistor. For this scenario, the difference in resistance incurred by the strain gauge was sufficiently small for the Analog Discovery to accommodate. However, the trend as visible from the plot implies that the first peak measure still may have been inaccurate (i.e. capped out) due to the limitations of the instrument. Fortunately, the experiment was not concerned with the immediate strain caused by the impact; the difference in voltage due to the appliance of weight can be calculated by the juxtaposing the equilibrium value taken from the time at which the weight settled (.4924 V) to the initial value (0.05237 V): this yields the value of 0.44 volts.

To figure out the change in resistance incurred by the strain gauge, the second equation was applied as follows:

```
>> S = @(G,V) -121*(G*(2*V+5)+500000) / (G * (2*V-5) - 500000);
disp(S(200,.44));
  121.0850
```

Here, S is R_{strain}, G is R_{gain}, and V is the measured voltage difference.
Subtracting the theoretical data of 121Ω from the value, we now obtain .0850Ω as the additional resistance supplied by the strain gauge.

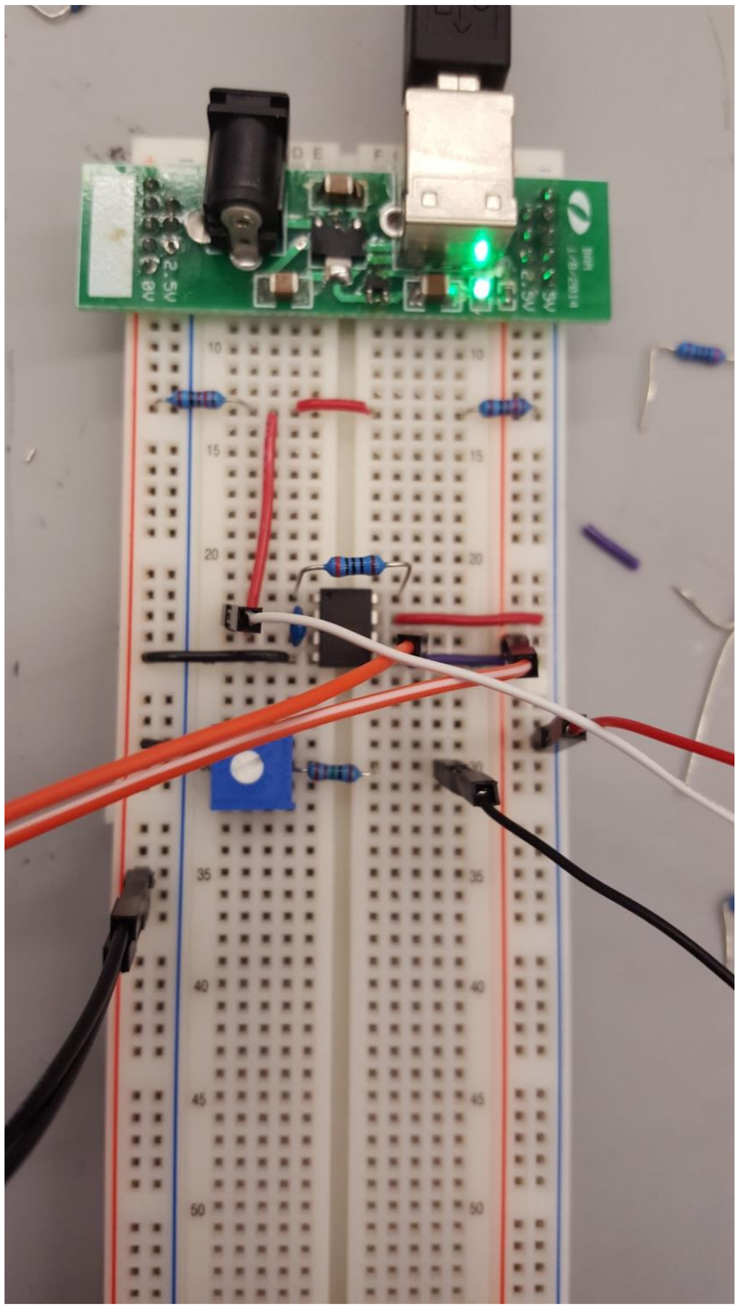
3)

The smallest voltage resolution obtainable from this circuit is – at least, by the Analog Discovery – 2 mV, or 0.002 V. Retaining the R_{gain} of 100Ω, the smallest resistance change can be found by applying the same equation:

```
>> S = @(G,V) -121*(G*(2*V+5)+500000) / (G * (2*V-5) - 500000);
disp(S(200,.002));
   121.0004
```

It is thus seen that the smallest detectible resistance change is .0004Ω.

4)



In order to legitimize the theoretical model of the RC circuit, experimental and theoretical data were plotted against each other:

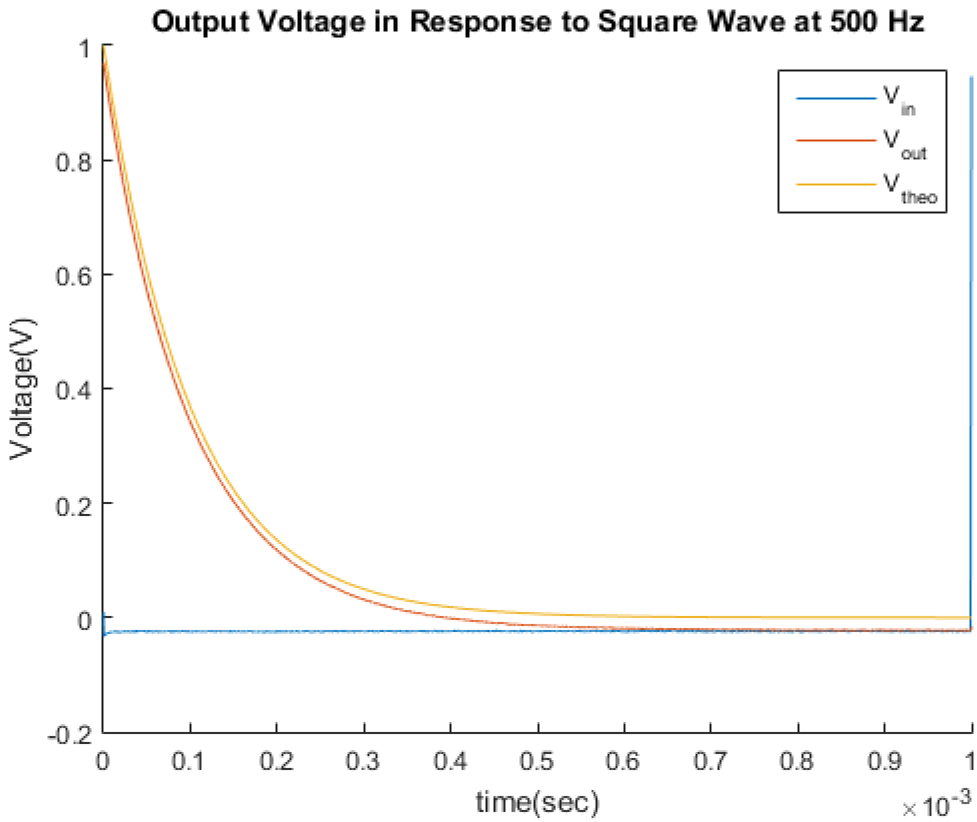


Figure 1. The graph of input, output, and theoretical voltage over 1 millisecond; although not apparent in the graph, V_{in} is tangent to the y axis at x=0.

As depicted above, the theoretical Voltage calculated from $V = e^{-t/(RC)}$ is a close approximation to the experimental value, with rather trivial deviation. This graph was implemented as follows:

```
t = linspace(0,.001);
R = 1000; %Ohms
C = 0.1 / 1000 / 1000; %farads
V_theo = exp(-t/R/C);
plot(t,V_theo);
```

It is thus verified that the output voltage, through the capacitor, adheres to the input voltage logarithmically: a result consistent with the anticipation.

Next, the resultant current was calculated from the respective values according to the relation $I = \Delta V/R$, where $\Delta V = V_{in} - V_{out}$ (experimental), or $-V_t$ (theoretical). Alternatively, for the theoretical model, solving $I = C * \frac{dV}{dt}$ yields an equivalent relationship, although the same technique cannot be applied to experimentally obtained data. The resultant plot is shown below:

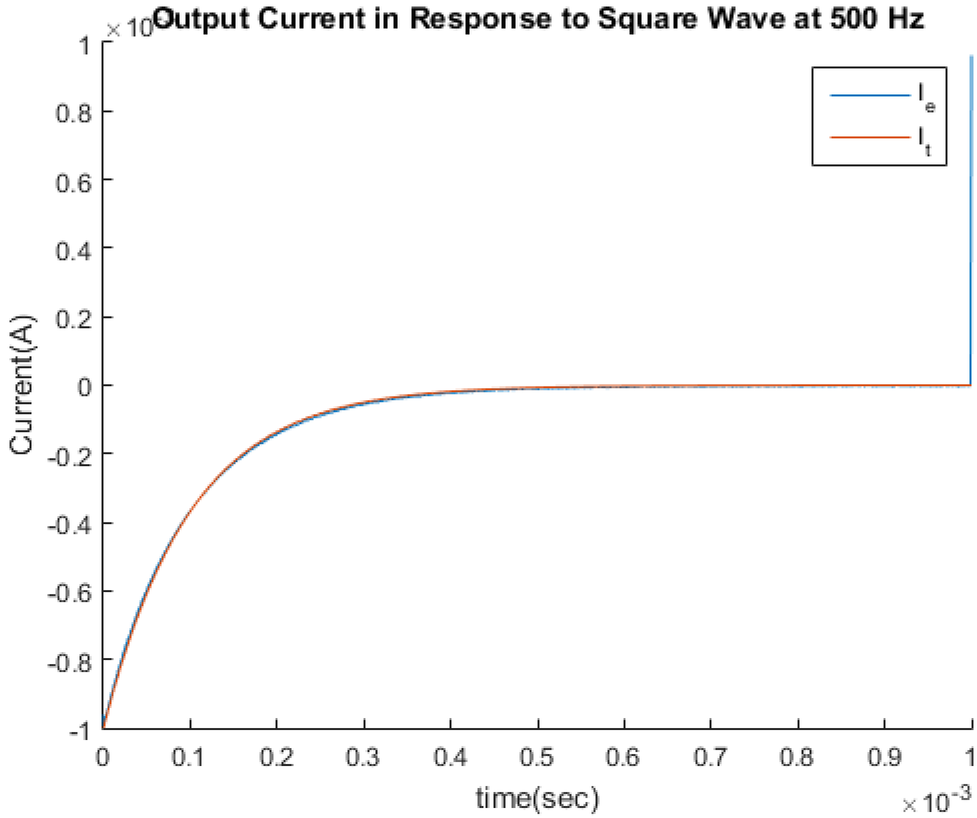


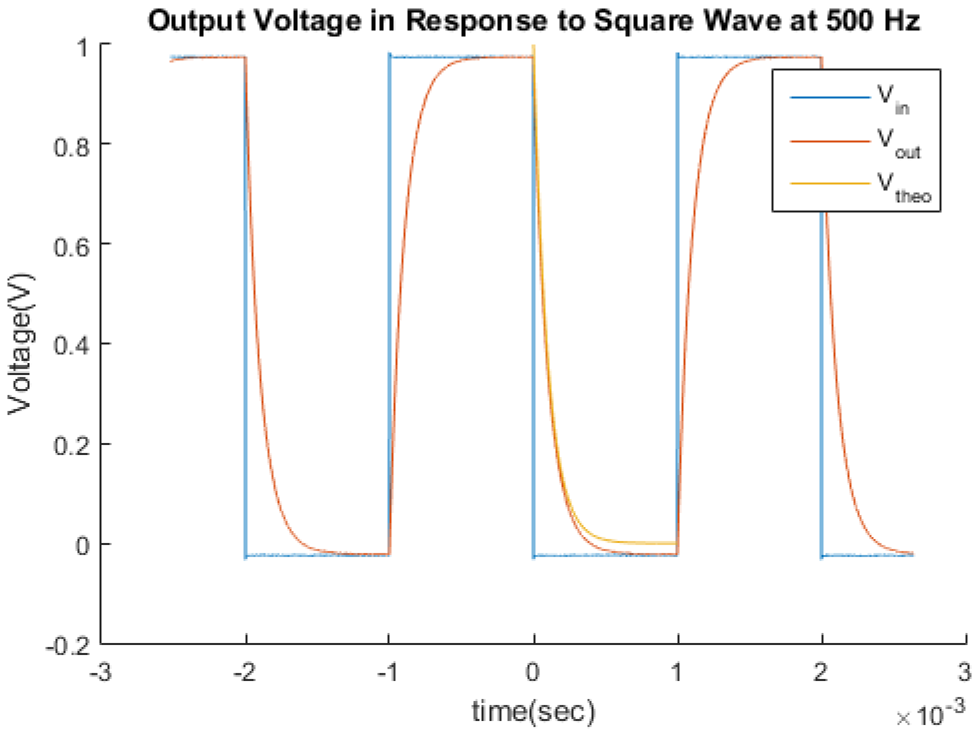
Figure 2. The current calculated from the voltage data obtained in the previous section.

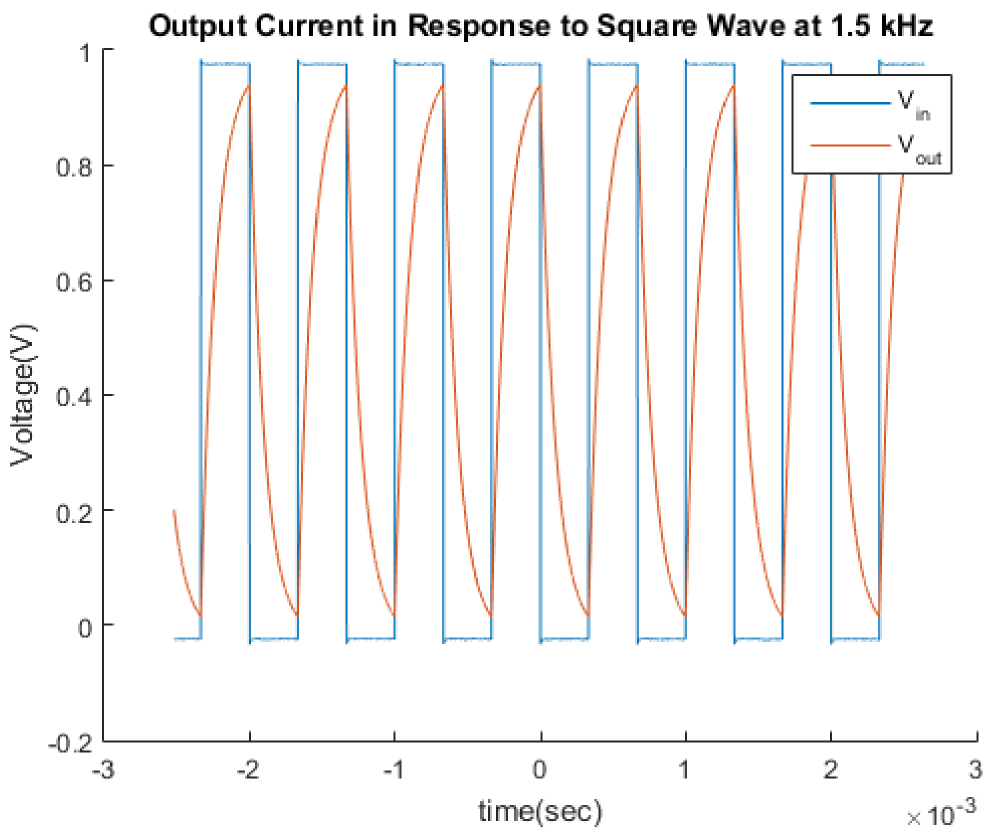
The two plots are nearly overlapping, which clearly illustrates that the theoretical model accurately describes the relationship. The following is its implementation, with variable values consistent with the aforementioned script.

```
I_e = (V_in - V_out)/ R;
I_t = -V_theo/R;

figure;
hold on;
plot(time,I_e);
plot(t, I_t);
```

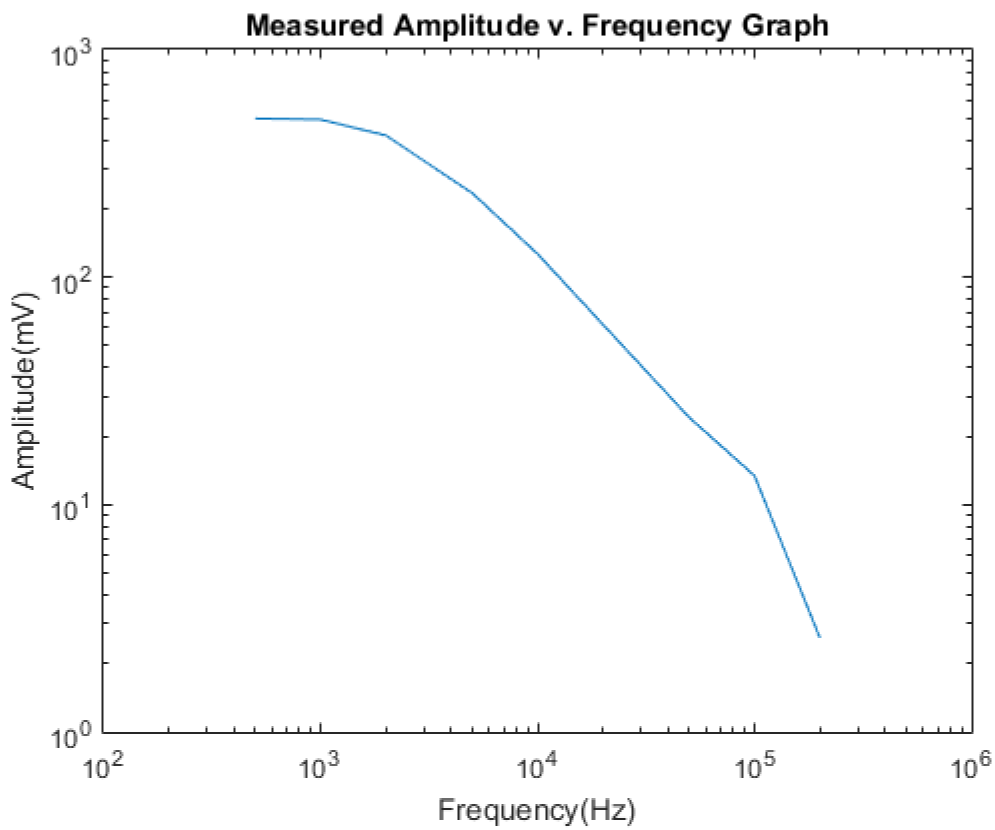
Now, to further emphasize the relationship between frequency, voltage and amplitude, the two graphs at 500Hz and 1.5kHz were plotted on the same scale applied to each axes:



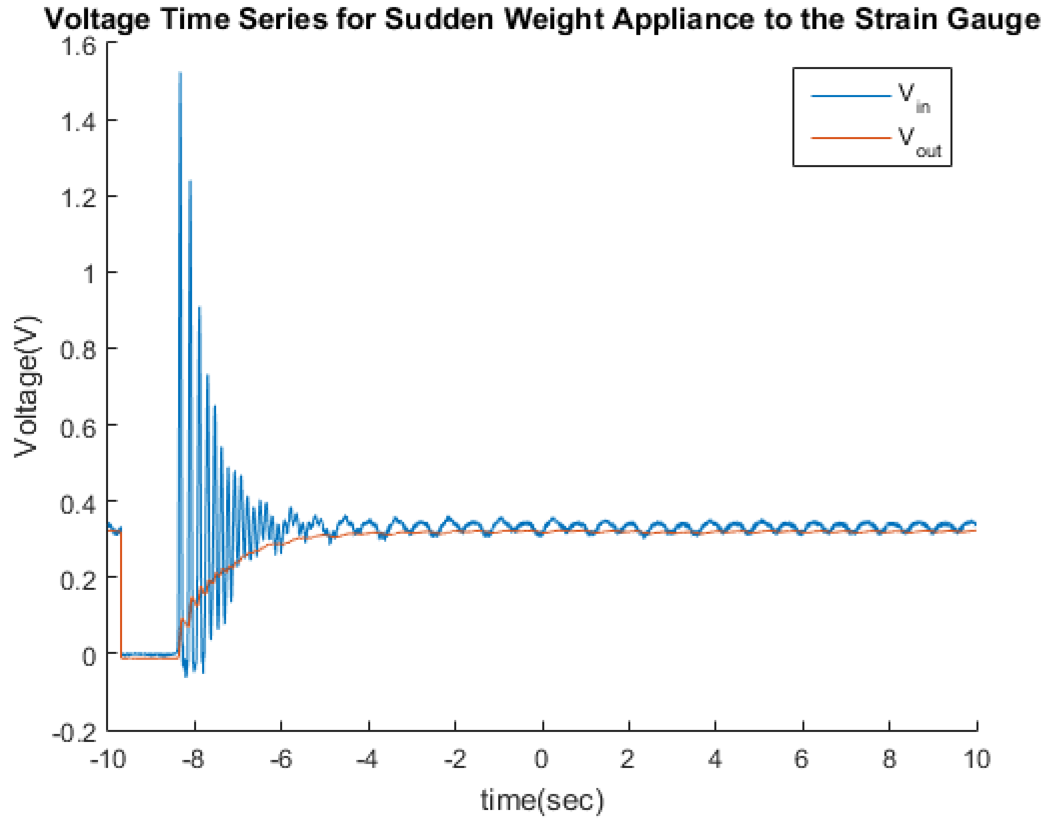


It is clear that the response of the output voltage at higher frequency – 1.5kHz – cannot quite reach the full amplitude, which is coherent with the model: the delay incurred upon the capacitor renders it unable to get fully charged.

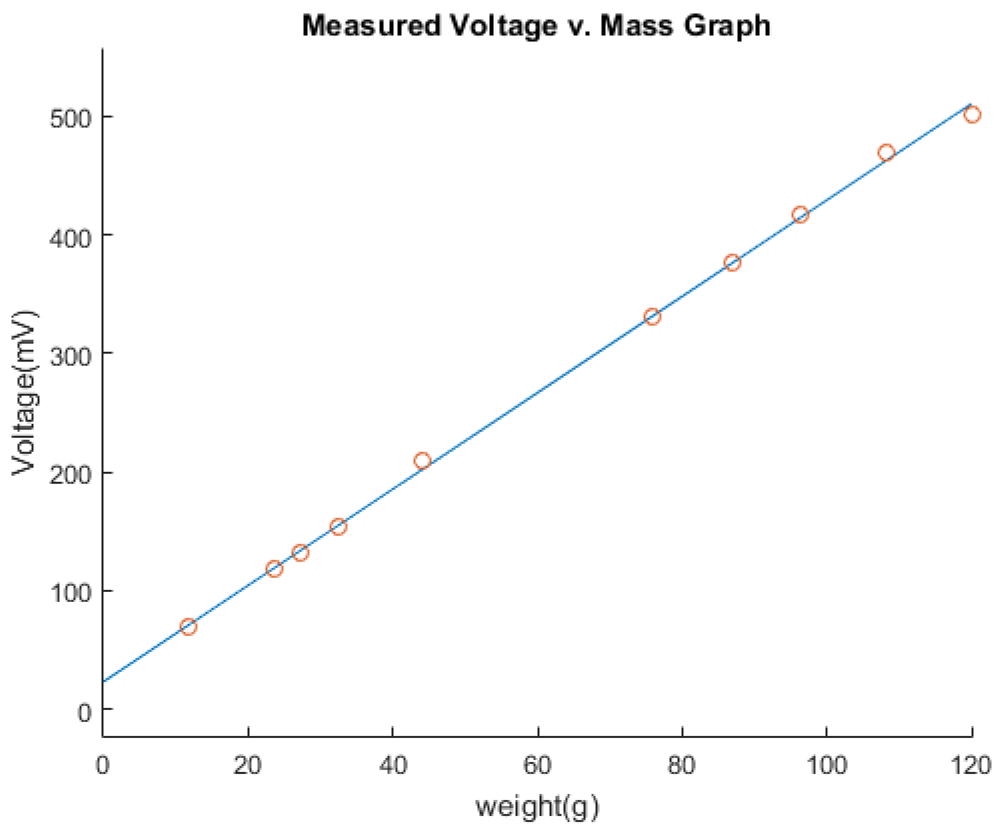
The characteristic frequency of the circuit is $1/(R * C * 2 * \pi)$ to convert from radians, which yields 1592 Hz. As the circuit is built in a low-pass filter scheme, 1500 Hz is under the cutoff frequency; therefore, its amplitude is relatively preserved from corruption, although its proximity to the cutoff frequency – and the imperfection of the filter – renders it not quite able to reach full amplitude.



This figure illustrates the steepening of the curve after the cutoff frequency, albeit with deviations and shortage in the measurements to the lower and higher registers: to elaborate, it can be seen that the graph begins to fall around the 1.6 kHz region, which is approximately the cutoff frequency.



The same pattern seen from the square-wave measurements reappear, though with much greater smoothing in the fluctuation of the curve. Because the curve is a diminishing sinusoidal wave with variant frequencies and amplitudes with respect to time, no quantitative conclusion can quite be drawn from the plot; the tendency of the capacitor to remain level to the rapid fluctuations persist, however, and provide insight that the circuit is still true to the principles we had hereto explored.



The best fit line was $4.06\, {}^{mV}\!/_{g} * \text{weight} + 22.9729\text{ mV}$, which indicates that the calibration value for mass at 0 grams is 22.9729 mV. Generally, the relationship is clearly defined as linear, and 4.06 mV will be administered per every gram. Since the Analog Discovery documentation states the device can measure down to 300 µV, this translates to .07 grams in terms of the smallest measurable gram value.

ISim Lab4: EKG

Yoonyoung Cho

October 1 2015

1 RC Filters

1.1 High-Pass Filter

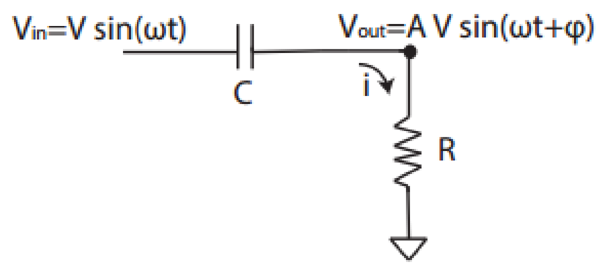


Figure 1: Circuit Diagram of a high-pass filter.

In a High-Pass Filter, the relationship between Frequency, Amplification, and Phase can be derived as follows:

$$V_{in} = V sin(\omega t) \tag{1}$$
$$V_{out} = AV sin(\omega t + \phi) \tag{2}$$
$$C\frac{d(V_{out} - V_{in})}{dt} = \frac{0 - V_{out}}{R} = I \tag{3}$$

This expands to:

$$C\frac{d(AV sin(\omega t + \phi) - V sin(\omega t))}{dt} = -\frac{AV sin(\omega t + \phi)}{R}$$
$$-RC(AV\omega cos(\omega t + \phi) - V\omega cos(\omega t)) = AV sin(\omega t + \phi)$$
$$-RC(A\omega cos(\omega t + \phi) - \omega cos(\omega t)) = Asin(\omega t + \phi)$$

Applying the trigonometric identities,

$$-RC(A\omega(cos(\omega t)cos(\phi) - sin(\omega t)sin(\phi)) - \omega cos(\omega t)) = A(sin(\omega t)cos(\phi) + cos(\omega t)sin(\phi)$$

As this equation must be true at all times, the terms with $sin(\omega t)$ and $cos(\omega t)$ must equate separately. Therefore,

$$RCA\omega(sin(\omega t)sin(\phi)) = Asin(\omega t)cos(\phi) \tag{4}$$
$$-RCA\omega(cos(\omega t)cos(\phi)) + RC\omega cos(\omega t)) = Acos(\omega t)sin(\phi) \tag{5}$$

equation (4) is now further simplified to yield the phase ϕ:

$$RCA\omega(sin(\omega t)sin(\phi)) = Asin(\omega t)cos(\phi)$$
$$RC\omega sin(\phi) = cos(\phi)$$
$$cot(\phi) = RC\omega$$

$$\phi = acot(RC\omega) \tag{6}$$

and equation (5) yields Amplification in terms of ω and ϕ.

$$-RCA\omega(cos(\omega t)cos(\phi)) + RC\omega cos(\omega t)) = Acos(\omega t)sin(\phi)$$
$$-RCA\omega cos(\phi) + RC\omega = Asin(\phi)$$
$$A(sin(\phi) + RC\omega cos(\phi)) = RC\omega$$

$$A = \frac{RC\omega}{sin(\phi) + RC\omega cos(\phi)} \tag{7}$$

As it is possible to sequentially solve for phase and amplification in MATLAB, there was no need to substitute ϕ and solve for the amplification expression. The resultant Bode plot for R = 1000 Ω, C = 0.1 µF is shown below:

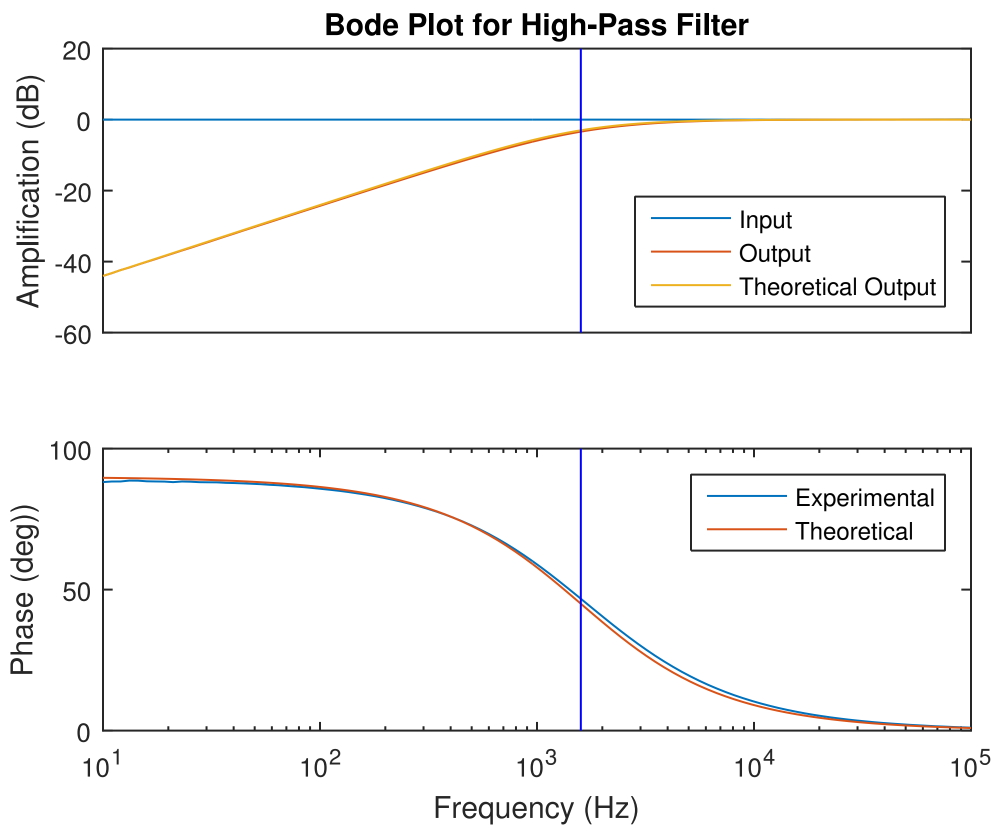


Figure 2: The Bode Plot for high-pass filter; the frequency ranges from 10Hz to 100kHz, and the blue line denotes the critical frequency, at 1592 Hz.

As shown, experimental and theoretical data are primarily consistent; the minor discrepancy in amplification and phase can be attributed to imperfections in the capacitor.

1.2 Low-Pass Filter

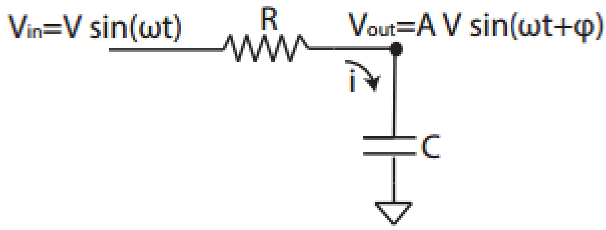


Figure 3: Circuit Diagram of a low-pass filter.

In a Low-Pass Filter, the relationship between Frequency, Amplification, and Phase can be derived as follows:

$$V_{in} = V sin(\omega t) \tag{8}$$
$$V_{out} = AV sin(\omega t + \phi) \tag{9}$$
$$C\frac{d(0 - V_{out})}{dt} = \frac{V_{out} - V_{in}}{R} = I \tag{10}$$

This expands to:

$$\begin{aligned} -C\frac{dV_{out}}{dt} &= \frac{V_{out} - V_{in}}{R} \\ -RC\frac{d(AV sin(\omega t + \phi))}{dt} &= AV sin(\omega t + \phi) - V sin(\omega t) \\ -RCAV\omega cos(\omega t + \phi) &= AV sin(\omega t + \phi) - V sin(\omega t) \\ -RCA\omega cos(\omega t + \phi) &= A sin(\omega t + \phi) - sin(\omega t) \end{aligned}$$

Applying the trigonometric identities,

$$-RCA\omega(cos(\omega t)cos(\phi) - sin(\omega t)sin(\phi)) = A(sin(\omega t)cos(\phi) + cos(\omega t)sin(\phi)) - sin(\omega t)$$

As this equation must be true at all times, the terms with $sin(\omega t)$ and $cos(\omega t)$ must equate separately. Therefore,

$$-RCA\omega(cos(\omega t)cos(\phi)) = Acos(\omega t)sin(\phi) \tag{11}$$
$$RCA\omega(sin(\omega t)sin(\phi)) = Asin(\omega t)cos(\phi) - sin(\omega t) \tag{12}$$

equation (11) is now further simplified to yield the phase ϕ:

$$\begin{aligned} -RCA\omega(cos(\omega t)cos(\phi)) &= Acos(\omega t)sin(\phi) \\ -RC\omega cos(\phi) &= sin(\phi) \\ -RC\omega &= \frac{sin(\phi)}{cos(\phi)} \\ tan(\phi) &= -RC\omega \end{aligned}$$

$$\phi = atan(-RC\omega) \tag{13}$$

and equation (12) yields Amplification in terms of ω and ϕ.

$$RCA\omega(sin(\omega t)sin(\phi)) = Asin(\omega t)cos(\phi) - sin(\omega t)$$
$$RCA\omega sin(\phi) = Acos(\phi) - 1$$
$$A(RC\omega sin(\phi) - cos(\phi)) = 1$$

$$A = 1/(RC\omega sin(\phi) - cos(\phi)) \tag{14}$$

The resultant Bode plot for R = 1000 Ω, C = 0.1 µF is shown below:

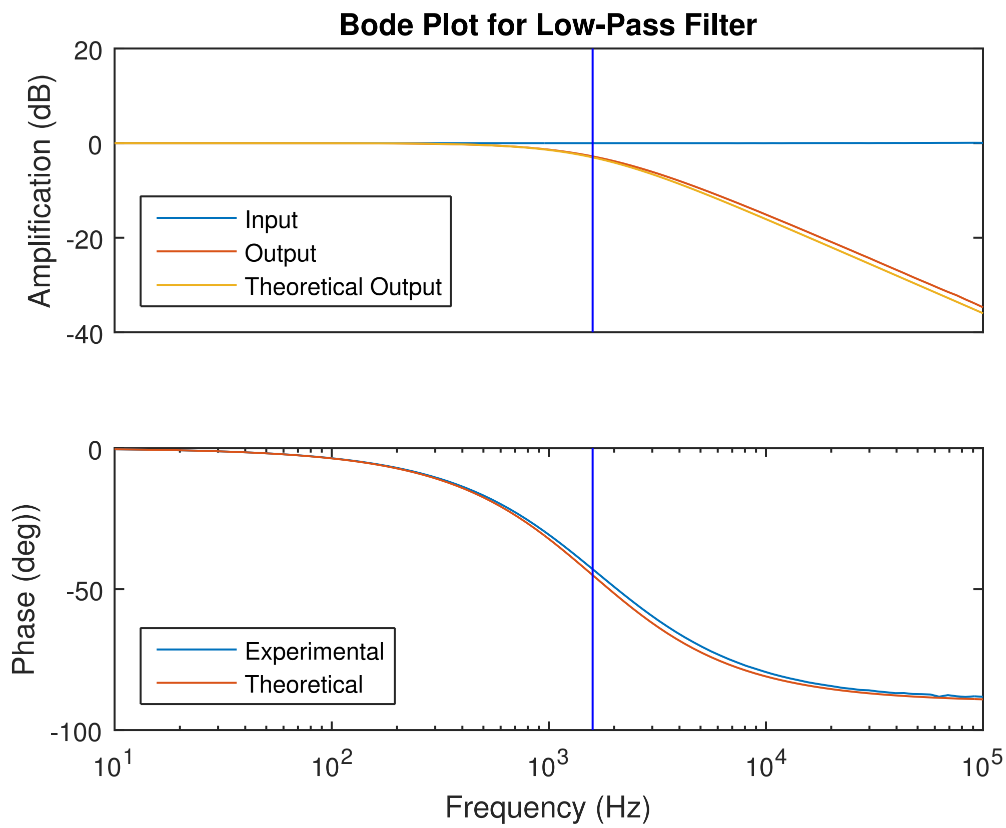


Figure 4: The Bode Plot for low-pass filter; the frequency ranges from 10Hz to 100kHz, and the blue line denotes the critical frequency, at 1592 Hz.

Such consistent coherence in the theoretical and experimental plots serve as sufficient verification of the theoretical model. The calculated cutoff frequency for both circuits was (since resistance and capacitance were held the same) $\frac{1}{1000\Omega * 0.1\,\mu F * 2\pi} = 1592Hz$, which is consistent to both of the plots.

1.3 Both Filters Applied

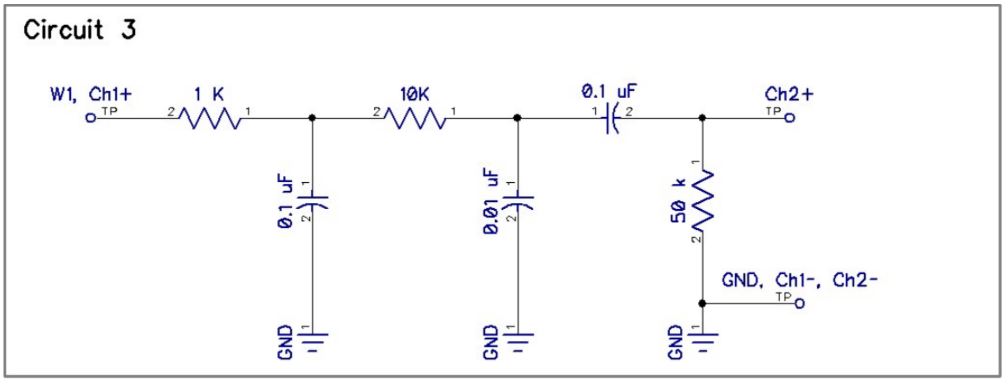


Figure 5: Circuit Diagram of two low-pass filters followed by a high-pass filter.

As for this scenario, in which multiple filters were applied to the source voltage consecutively, amplification was multiplied and phase was added per each filter, which is a natural assumption given the type of transformations they undergo in filtration. The resultant Bode plot which adheres to the above diagram is shown below:

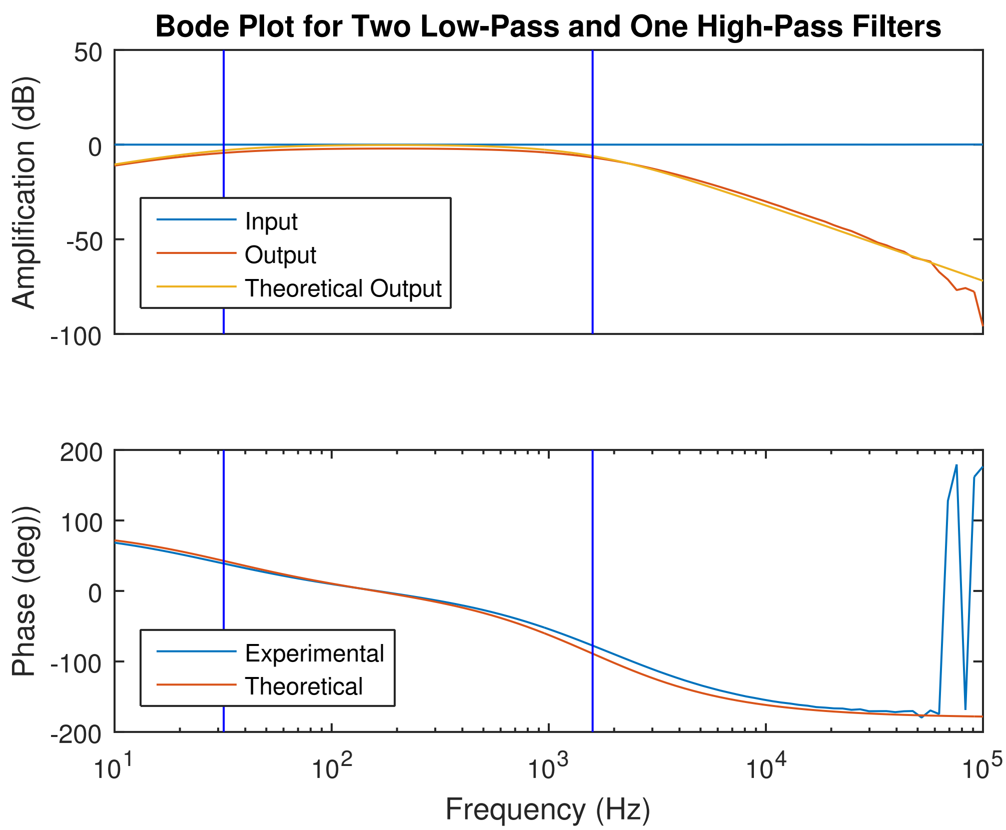


Figure 6: The Bode Plot for a combination of two low-pass filters and one high-pass filter, the frequency ranges from 10Hz to 100kHz. The blue line denotes the critical frequency, at 31.82Hz and 1592 Hz.

The irregularities shown an the end of the experimental plot are due to the confusion between 180° and -180°, as well as the inability for the analog discovery to measure in such fine resolutions.

2 The EKG

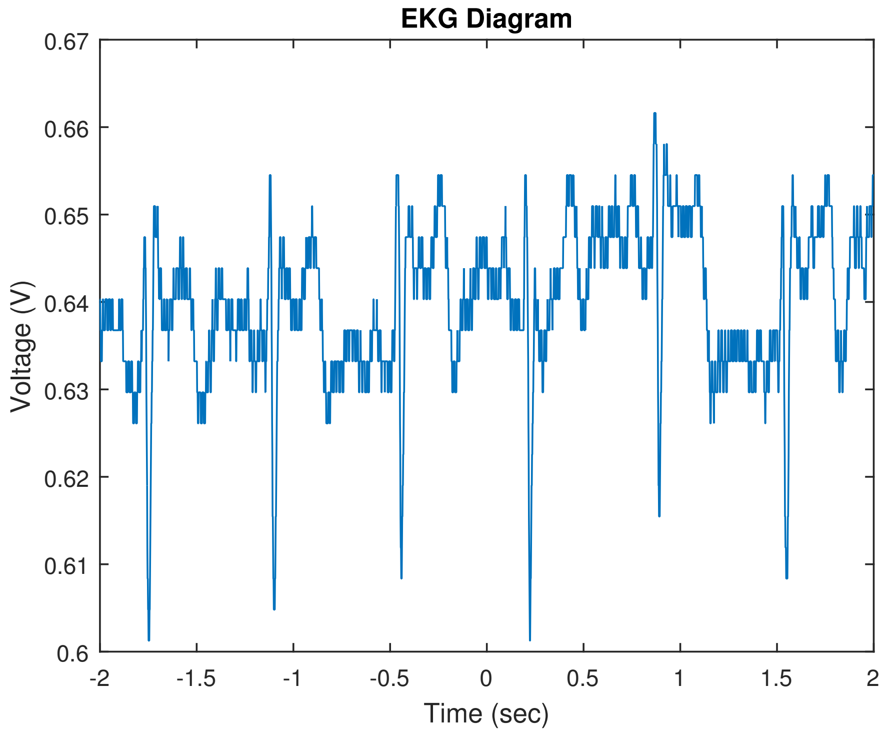


Figure 7: EKG Diagram

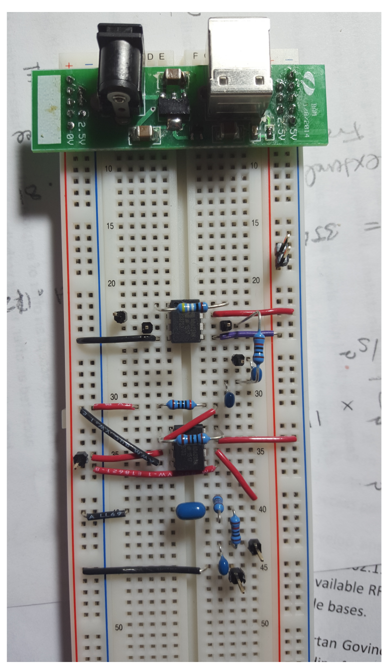


Figure 8: Circuit Picture

ISim Lab 5: Op Amps

Yoonyoung Cho

October 7 2015

1 Op Amp as Buffer

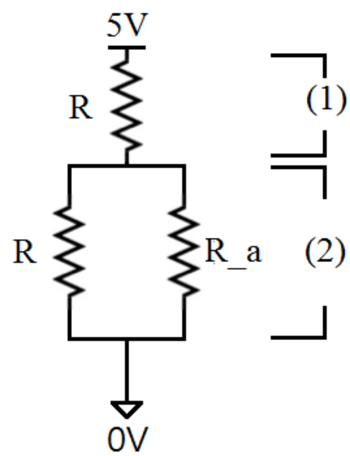


Figure 1: Simplified Circuit Diagram; Analog Discovery is represented as a resistor with resistance R_a.

Given:

$$R_1 = R$$
$$R_2 = \frac{1}{\frac{1}{R} + \frac{1}{R_a}}$$
$$I_1 = V_1/R_1$$
$$I_2 = V_2/R_2$$
$$I_1 = I_2$$

Yields

$$R_2 = \frac{V_2 R}{V_1} = \frac{1}{\frac{1}{R} + \frac{1}{R_a}}$$
$$\frac{V_2 R}{V_1} = \frac{R_a * R}{R_a + R}$$
$$\frac{V_2}{V_1} = \frac{R_a}{R_a + R}$$
$$R_a = \frac{R * V_2}{V_1 - V_2}$$

Thus the impedence of the Analog Discovery is $\frac{1M\Omega * 1.73V}{3.27V - 1.73V} = 1.06M\Omega$.

Accordingly, the theoretical measured Voltage with $499K\Omega$ Resistors is:

$$\begin{aligned}\frac{R}{R_a} &= \frac{V_1 - V_2}{V_2} \\ &= \frac{5 - 2 * V_2}{V_2} \\ R * V_2 &= R_a(5 - 2 * V_2) \\ (R + 2 * R_a)V_2 &= 5R_a \\ V_2 &= \frac{5R_a}{R + 2R_a}\end{aligned}$$

So $V_2 = \frac{5V*1.06M\Omega}{.499M\Omega+2*1.06M\Omega} = 2.02V$.

Table 1: Voltage measurement through experiment

Resistance	Op Amp	Voltage
1 MΩ	X	1.73 V
499 KΩ	X	2.04 V
499 KΩ	O	2.51 V

As shown, the experimental value of 2.04V, when compared to the theoretical value of 2.02V, yields a % discrepancy of -0.99%. Meanwhile, the same circuit with the Op Amp demonstrated much closer proximity to the anticipated value (2.51 V), which evidences its capacity as a buffer. In the past labs, the value of R was small enough such that the value of R_2 approximated closely to R; in this experiment, we adopted a $1M\Omega$ resistor, which is on the same order of magnitude as the impedence of the Analog Discovery; hence, the voltage drop through the Analog Discovery was noticeably significant.

2 Inverting Amplifier

Assuming that no current goes through the Op Amp, the circuit is equivalent to the following diagram:

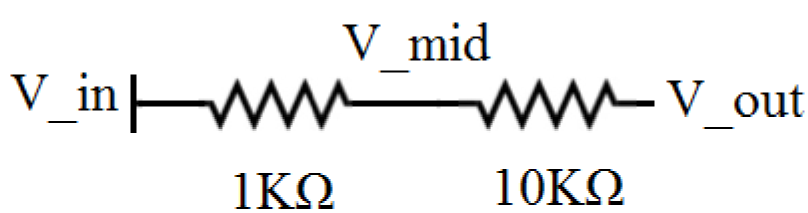


Figure 2: Simplified Circuit Diagram. When V_{out} is not at its extremes, V_{mid} is 2.5V when $V_{pos} = V_{neg}$; in reference to 2.5V, thus, it will be considered zero.

Since the circuit is congruent to that of a voltage-divider, it can be easily deduced that when V_{mid} is equivalent to $\frac{V_{mid}-V_{in}}{1K\Omega} = \frac{V_{out}-V_{mid}}{10K\Omega}$, and since $V_{mid} = 0$,

$$V_{out} = -10V_{in}$$

The output voltage will thus be amplified by 10. The below figure illustrates this relationship:

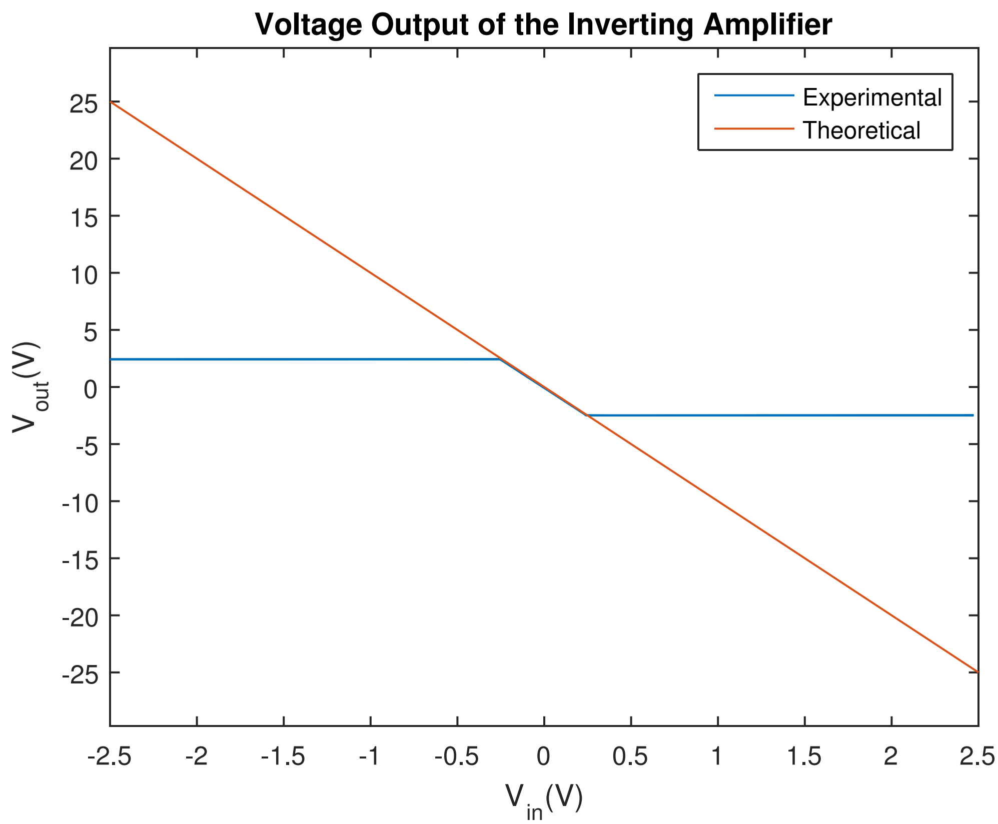


Figure 3: The relationship between input and output voltage through the inverting amplifier.

The theoretical line was generated simply by setting $V_{out} = -10V_{in}$; as seen, the slope of the line (where V_{out} is not limited by the rails) coheres closely to -10; in other places, the value is fixed at the rails.

3 Op Amp Filter

3.1 Approach with Resistance and Capacitance

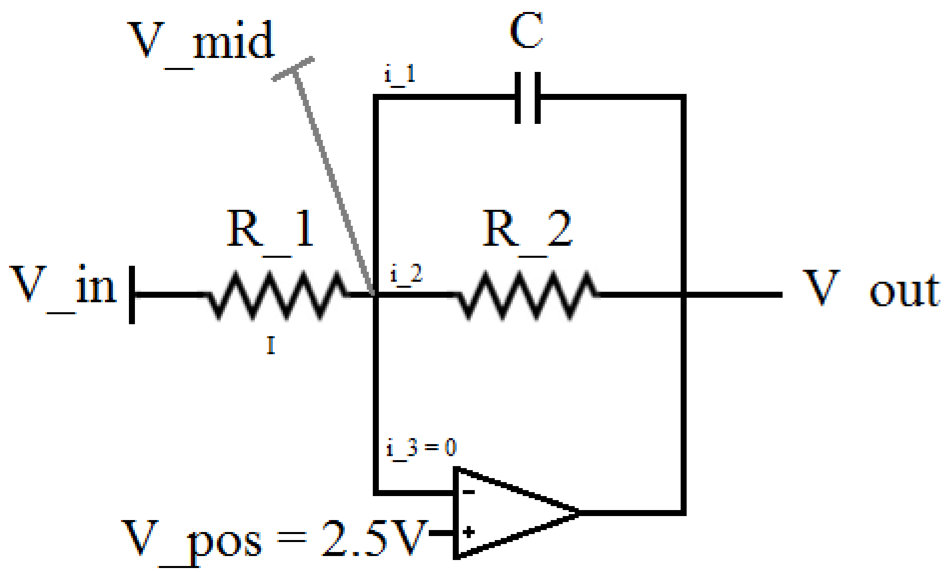


Figure 4: The circuit diagram of the Op-Amp Filter, which will be extensively referred to in the following section.

Since the voltage across the capacitor and the second resistor should be the same and the addition of the two currents, the relationship can be sumamrized as follows:

$$I = i_1 + i_2 \tag{1}$$

$$I = \frac{V_{mid} - V_{in}}{R_1} \tag{2}$$

$$i_1 = \frac{V_{out} - V_{mid}}{R_2} \tag{3}$$

$$i_2 = C\frac{d(V_{out} - V_{mid})}{dt} \tag{4}$$

Now, although V_{in} is actually $2.5 + 0.1sin(\omega t)$, taking the voltage in reference to V_{pos}, V_{in} can be represented as $0.1sin(\omega t)$. Accordingly, V_{mid} is also 0 if V_{out} $!= -2.5V$ and V_{out} $!= 2.5V$[1], in which case it is stuck at the 'rails' and the assumption that $V_{pos} == V_{neg}$ no longer holds true. Furthermore, V_{out} will be modeled

[1]The maximum/minimum values of the Op-Amp, not represented in the diagram for simplification.

as $A * 0.1 * sin(\omega t + \phi)$; the objective is to identify Amplitude(A) and Phase(ϕ).

$$\begin{aligned}
\frac{V_{mid} - V_{in}}{R_1} &= \frac{V_{out} - V_{mid}}{R_2} + C\frac{d(V_{out} - V_{mid})}{dt} \\
\frac{0 - 0.1sin(\omega t)}{R_1} &= \frac{0.1Asin(\omega t + \phi) - 0}{R_2} + C\frac{d(0.1Asin(\omega t + \phi) - 0)}{dt} \\
-\frac{0.1sin(\omega t)}{R_1} &= \frac{0.1Asin(\omega t + \phi)}{R_2} + C\frac{d(0.1Asin(\omega t + \phi))}{dt} \\
-\frac{sin(\omega t)}{R_1} &= \frac{Asin(\omega t + \phi)}{R_2} + C\frac{d(Asin(\omega t + \phi))}{dt} \\
-\frac{sin(\omega t)}{R_1} &= \frac{Asin(\omega t + \phi)}{R_2} + CA\omega cos(\omega t + \phi)
\end{aligned}$$

now, invoking trigonometric identities,

$$-\frac{sin(\omega t)}{R_1} = \frac{A}{R_2} * (sin(\omega t)cos(\phi) + cos(\omega t)sin(\phi)) + CA\omega(cos(\omega t)cos(\phi) - sin(\omega t)sin(\phi))$$

For this equation to be true at all times, $sin(\omega t)$ and $cos(\omega t)$ terms must balance separately:

$$0 = \frac{A}{R_2}cos(\omega t)sin(\phi) + CA\omega cos(\omega t)cos(\phi) \tag{5}$$

$$-\frac{sin(\omega t)}{R_1} = \frac{A}{R_2} * sin(\omega t)cos(\phi) - CA\omega sin(\omega t)sin(\phi) \tag{6}$$

in order to obtain ϕ, equation (5) simplifies to:

$$\begin{aligned}
0 &= \frac{A}{R_2}cos(\omega t)sin(\phi) + CA\omega cos(\omega t)cos(\phi) \\
0 &= \frac{sin(\phi)}{R_2} + C\omega cos(\phi) \\
-C\omega cos(\phi) &= \frac{sin(\phi)}{R_2} \\
-R_2C\omega &= \frac{sin(\phi)}{cos(\phi)}
\end{aligned}$$

thus,

$$\phi = atan(-R_2C\omega) \tag{7}$$

in order to obtain Amplitude, equation (6) simplifies to:

$$\begin{aligned}
-\frac{sin(\omega t)}{R_1} &= \frac{A}{R_2} * sin(\omega t)cos(\phi) - CA\omega sin(\omega t)sin(\phi) \\
-\frac{1}{R_1} &= \frac{A}{R_2} * cos(\phi) - CA\omega sin(\phi) \\
-\frac{1}{R_1} &= A * (\frac{1}{R_2} * cos(\phi) - C\omega sin(\phi))
\end{aligned}$$

thus,

$$A = \frac{1}{R_1 * (C\omega sin(\phi) - \frac{1}{R_2} * cos(\phi))} \tag{8}$$

3.2 Approach with Impedence

Alternatively, this circuit could be solved by using the concept of impedence, simply replacing resistors and capacitors with an equivalent "impedor":

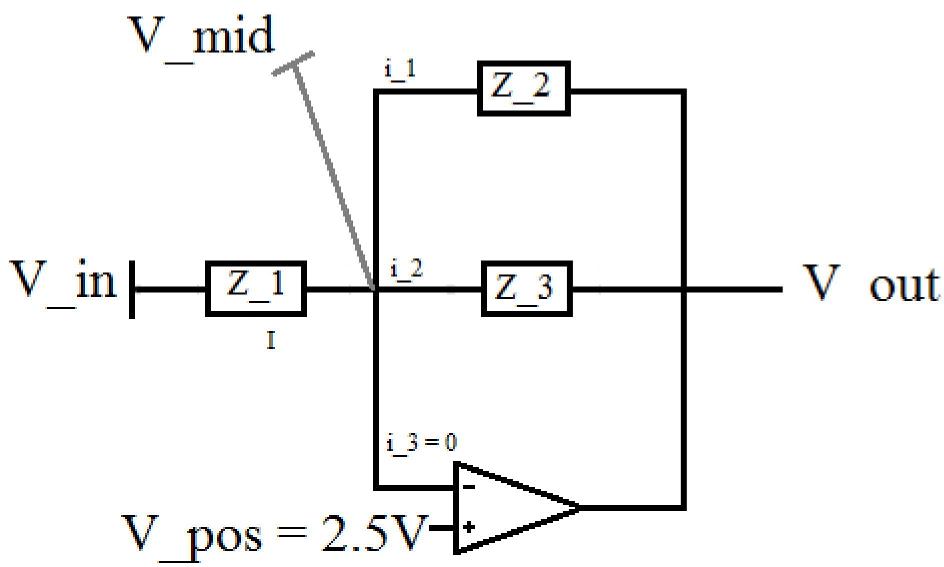


Figure 5: Caption

this circuit yields the following relationship:

$$\begin{aligned}
\frac{V_{mid} - V_{in}}{Z_1} &= \frac{V_{out} - V_{mid}}{\frac{1}{\frac{1}{Z_2} + \frac{1}{Z_3}}} \\
\frac{0 - V_{in}}{Z_1} &= \frac{V_{out} - 0}{\frac{1}{\frac{1}{Z_2} + \frac{1}{Z_3}}} \\
\frac{V_{in}}{Z_1} &= \frac{V_{out}}{\frac{1}{\frac{Z_2+Z_3}{Z_2 Z_3}}} \\
\frac{V_{in}}{Z_1} &= V_{out}\frac{Z_2 + Z_3}{Z_2 Z_3} \\
\frac{V_{out}}{V_{in}} &= \frac{Z_2 Z_3}{Z_1(Z_2 + Z_3)}
\end{aligned}$$

Since $Z_1 = 1\,\mathrm{k\Omega}$, $Z_2 = 1/j\omega C$, where C = 0.01 µF and $Z_3 = 1\,\mathrm{k\Omega}$,

$$\frac{V_{out}}{V_{in}} = \frac{1/(j\omega * 0.01\,\mu\mathrm{F}) * 1\,\mathrm{k\Omega}}{1\,\mathrm{k\Omega} * (1/(j\omega * 0.01\,\mu\mathrm{F}) + 1\,\mathrm{k\Omega})} \tag{9}$$

This yields a direct relationship of Amplitude and Phase to the frequency of V_{in}; the resultant Bode plot is shown below.

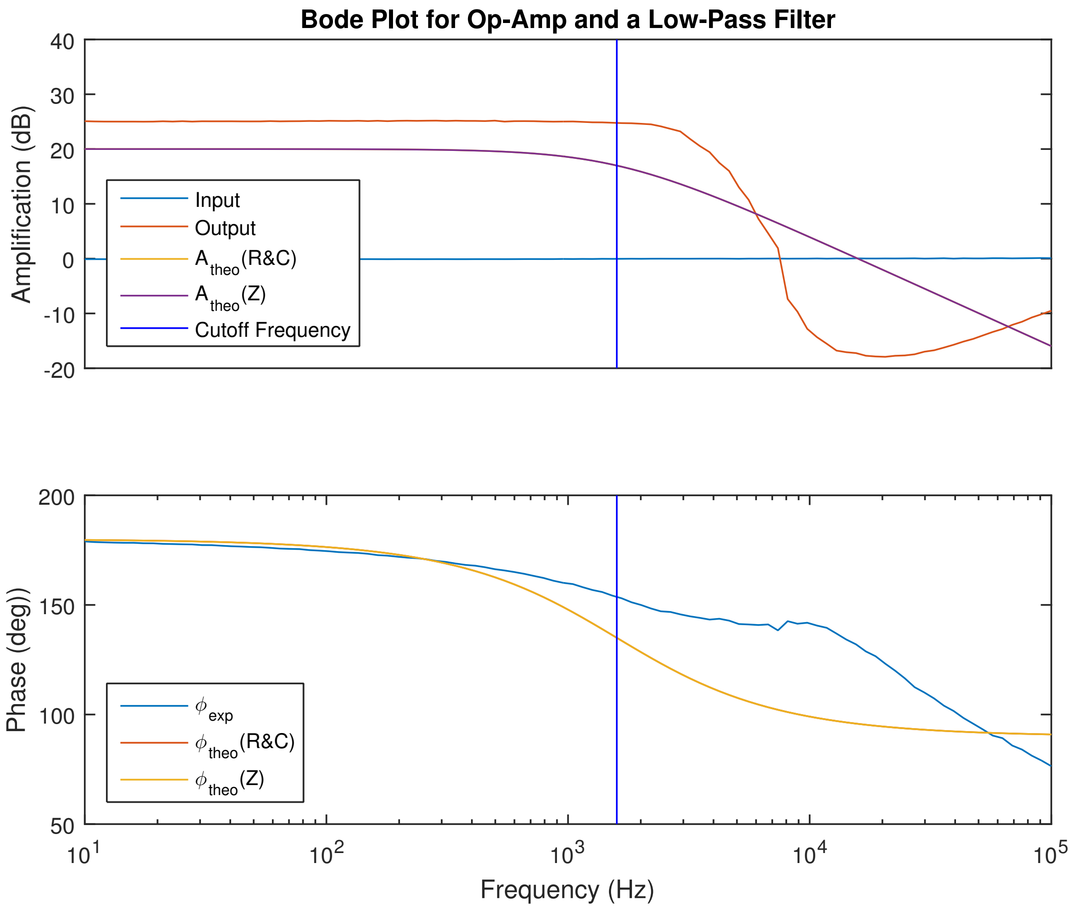


Figure 6: The Bode Plot of the circuit;paranthesized R & C indicates that the calculation was conducted by an analysis of resistance and capacitance; Z, on the other hand, indicates that the calculation was done with impedence. As seen, the two theoretical calculations exactly overlap.

while the results do not exactly match up, the two theoretical calculations are congruent; therefore, it is unlikely that the calculation themselves are both wrong. I would attribute the discrepancy between the two results (although they are indeed quite similar) to the limitations in the precision of devices, such as capacitors, as well as the ability of the analog discovery to pick up signals at high frequencies, as the discrepancy was more readily apparent in the higher frequency range.

4 Light Measurement

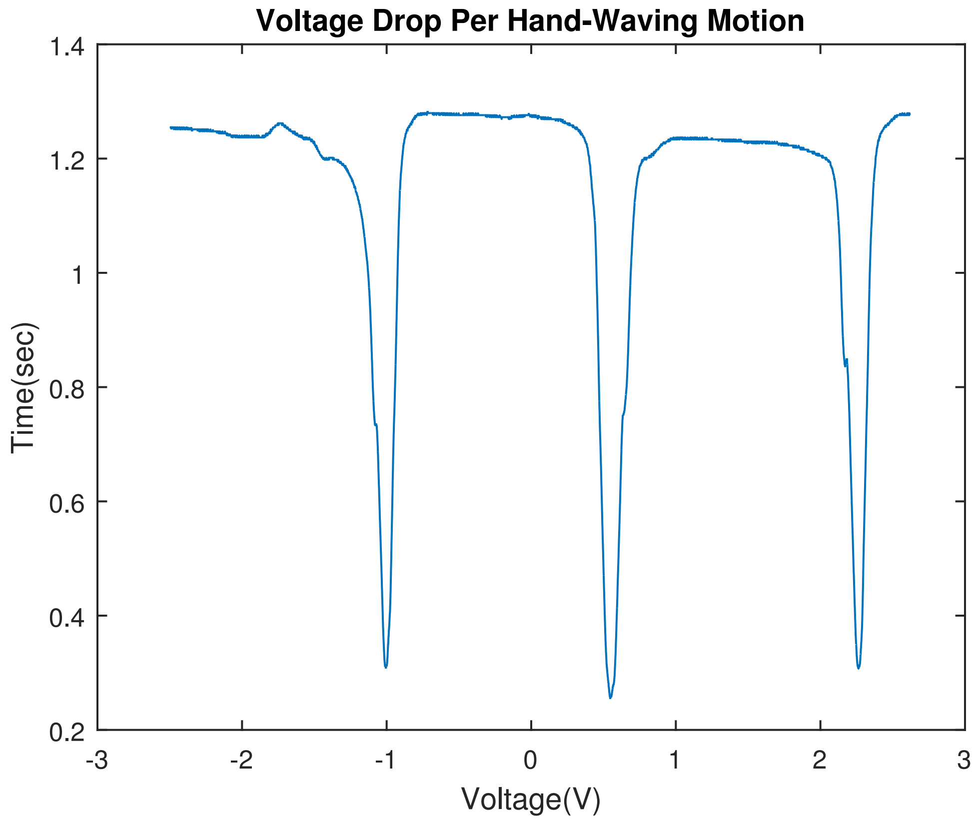


Figure 7: The troughs in the plot occurred when I waved my hand over the photodiode, which makes sense since it lost the supply of light, thereby reducing the electric potential. As soon as my hand was out of the way from the ambient light source, the voltage quickly recovered.

5 Pulse Measurement

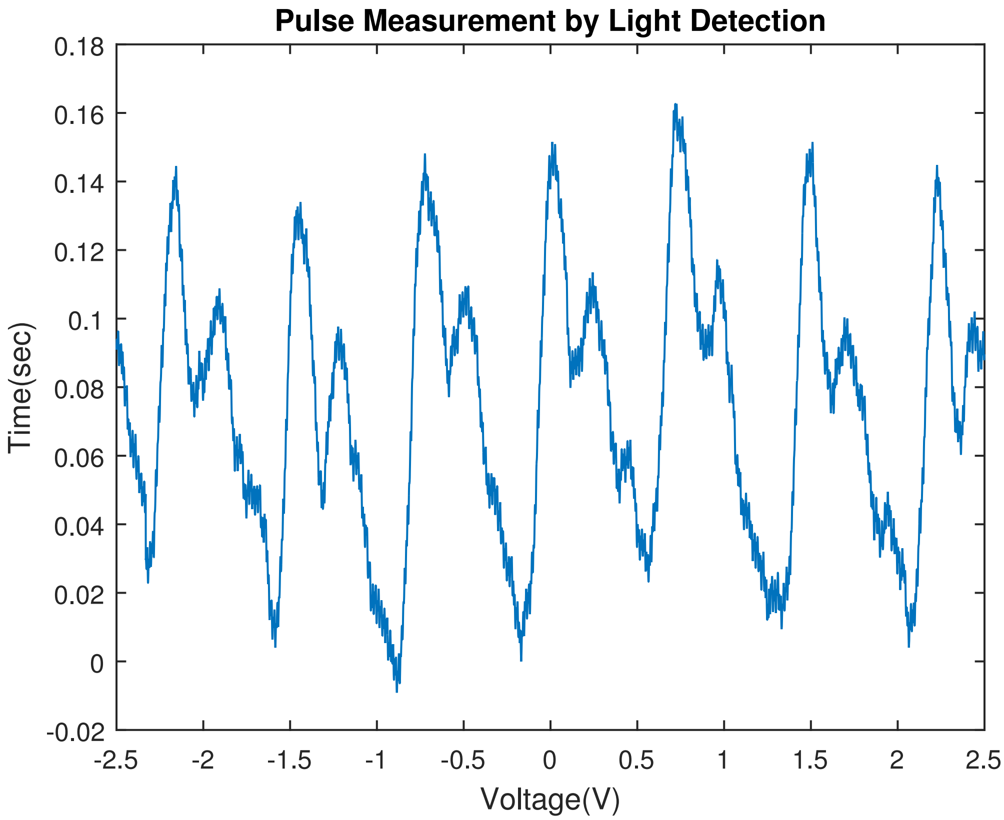


Figure 8: The Pulse, as measured by the intensity of light passing through the finger.

6 Photo

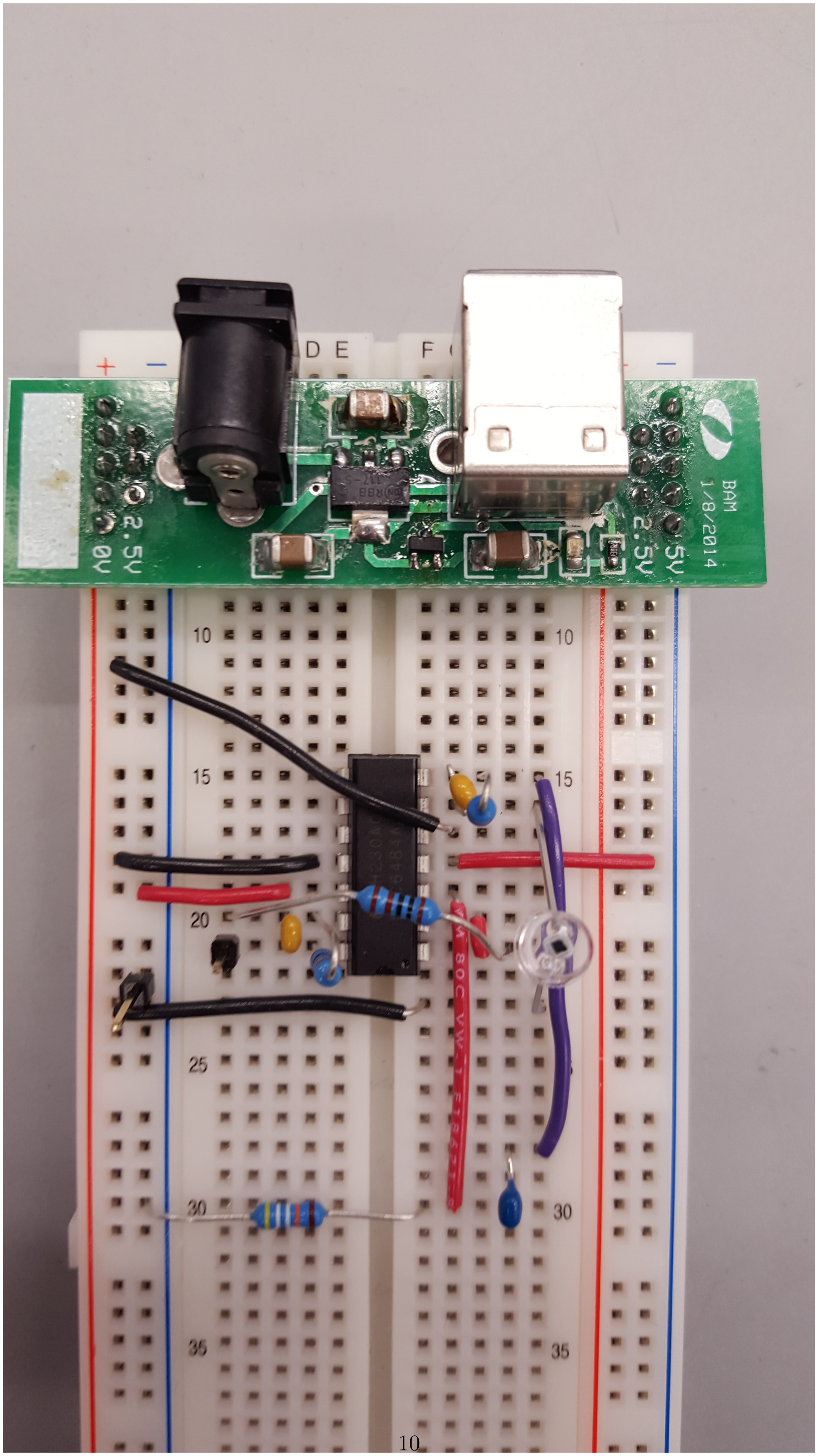


Figure 9: The picture of the pulse-measuring circuit.

ISim Lab 6 : Glucose Meter

Yoonyoung Cho

October 18 2015

1 Voltage Source

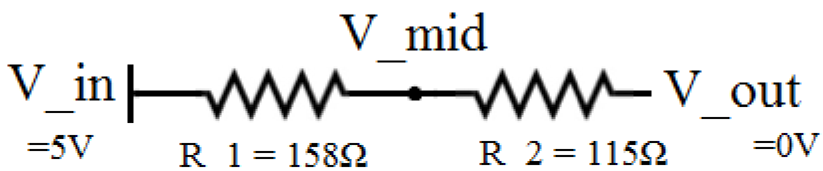


Figure 1: A simplified diagram of the circuit; the Op-Amp is not represented in the diagram, as it is assumed not to draw any current.

A quick analysis of the circuit shows that:

$$\frac{V_{mid} - V_{in}}{R_1} = \frac{V_{out} - V_{mid}}{R_2}$$
$$\frac{2.1V - 5V}{R_1} = \frac{0 - 2.1V}{R_2}$$
$$\frac{R_1}{R_2} = \frac{2.9V}{2.1V}$$

since $\frac{2.9V}{2.1V} = 1.381$ and $\frac{158\Omega}{115\Omega} = 1.374$, which shows a mere 0.5% deviation, I used the two resistors; the corresponding circuit provided the Voltage of $2.104V$ as the value for V_{mid}, consistent with the prediction.

2 Resistance Measure

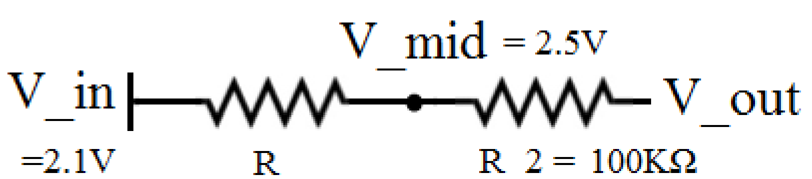


Figure 2: A simplified diagram of the circuit; the Op-Amps are, again, not represented, for simplicity. However, the values of V_{in} and V_{out} are affixed at $2.1V$ and $2.5V$, respectively, due to the Op-Amps.

Since the current through the two resisters ought to be equivalent,

$$\frac{V_{mid} - V_{in}}{R} = \frac{V_{out} - V_{mid}}{R_2}$$
$$R = R_2 * \frac{V_{mid} - V_{in}}{V_{out} - V_{mid}}$$

which, in this case, yields $100K\Omega * \frac{(2.5V - 2.1V)}{(3.26V - 2.5V)} = 52.6K\Omega$. As the prescribed value for the resistor was $50K\Omega$, the % Discrepancy of the two is a mere 5.2%. The following table summarizes the accuracy – and the overall characteristic – of this circuit:

Table 1: Actual(R_a) vs. Theoretical(R_t) Resistance

V_{out}	R_a	R_t
$3.306V$	$49K\Omega$	$49.63K\Omega$
$4.526V$	$20K\Omega$	$19.74K\Omega$
$4.974V$	$2K\Omega$	$16K\Omega$
$4.974V$	0Ω	$16K\Omega$
$2.572V$	$499K\Omega$	$555K\Omega$

It is readily apparent that the values are most coherent in the range that is within a modest leeway from $100K\Omega$; the values taken at extremity are less credible and demonstrate greater deviation from the actual values of the resistors.

For instance, a quick calculation of $R_{low} = \frac{(2.5V - 2.1V) * 100K\Omega}{5.0V - 2.5V}$ proves that the resistor value lower than $16K\Omega$, the point at which V_{out} gets stuck in the rails of the Op-Amp, cannot be expected to be consistent with the theoretical values, which is indeed the case for the measurements taken at $2K\Omega$ and 0Ω(wired connection without a resistor).

As this data is closely coherent to such predictions – displaying "bound" behavior at extremity and more consistency around the $100K\Omega$ region – I can conclude that the circuit is functional.

3 Integrator

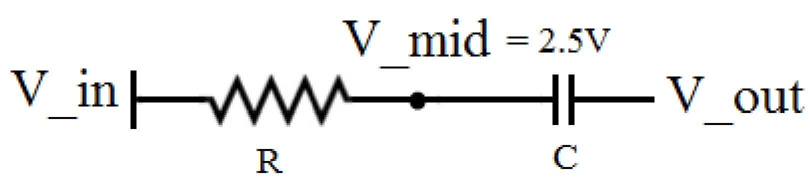


Figure 3: A simplified diagram of the circuit; V_{mid} is held to be at $2.5V$ due to the Op-Amp.

The relationship in this circuit is quite straightforward:

$$\frac{V_{mid} - V_{in}}{R} = C\frac{d(V_{out} - V_{mid})}{dt}$$

taking V_{out} as reference, the equation can be further simplified:

$$\frac{-V_{in}}{R} = C\frac{dV_{out}}{dt}$$

$$\int_0^t \frac{-V_{in}}{R}dt = \int_0^t CdV_{out}$$

$$V_{out} - V_{out_0} = \frac{\int_0^t -V_{in}dt}{RC}$$

Now, since V_{in}, is a square wave, which can be assumed constant over a brief time span,

$$V_{out} - V_{out_0} = \frac{-V_{in} * t}{RC}$$

With such relationship in mind, let us consider the following plot, generated from the experiment:

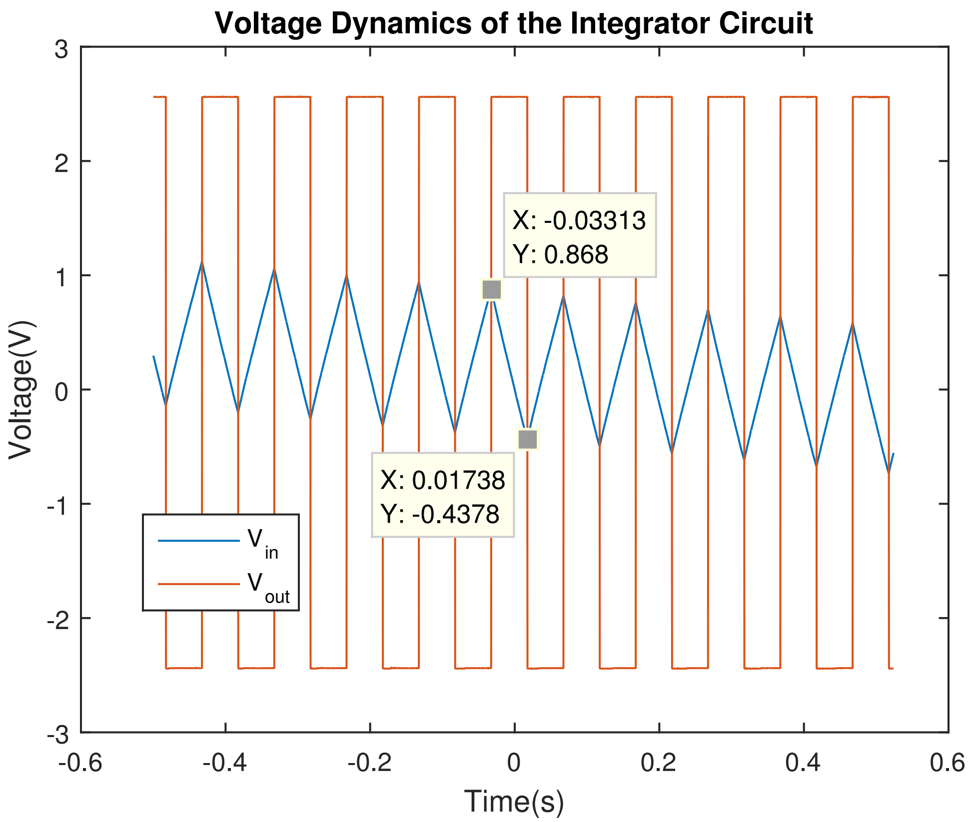


Figure 4: The Voltage Dynamics of the Integrator Circuit. The Slope is present in the graph because the snapshot was taken before the balance settled down; this choice was inevitable, as the integration was cut off prematurely by the -2.5V limit rails when the equilibrium state was reached.

Here, Δt is $0.01738 - (-0.03313) = .05051$ seconds, and $\Delta V_{out} = -0.4378 - 0.868 = -1.3058$. Theoretically, since $\Delta t = 0.05051$ seconds, $\Delta V_{out} = -2.5 * .05051/(100K\Omega * 1\,\mu\text{F})$; so the theoretical $\Delta V_{out} = -1.263V$, which is only 3.4% away from the experimental value. This coherence demonstrates that the circuit is in fact an integrating circuit.

4 Glucose Sensor

The superimposed plot of the Voltage(which is proportional to the current) over time is shown below:

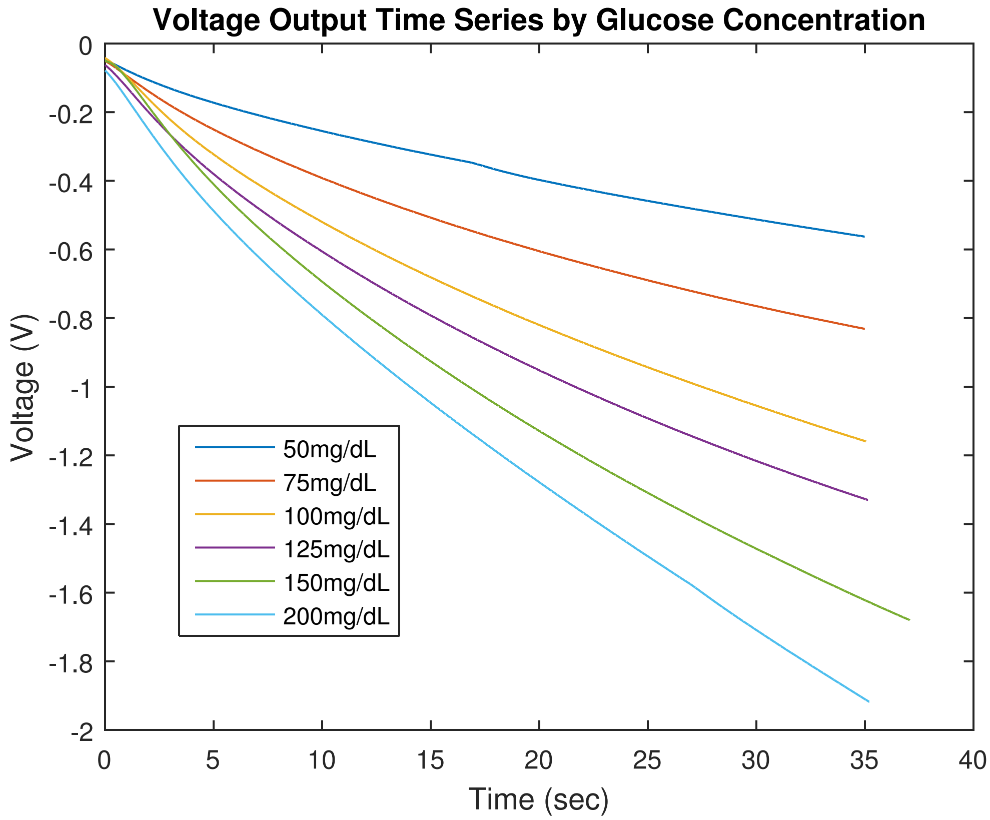


Figure 5: Voltage output time series. In superimposing the respective plots, the data was processed so that the plateaued region would be eliminated; thereby, each interaction would start at the same moment, giving a better idea of calibration.

as the trends reveal, the concentrations diverge at a slope that is approximately proportional to the concentration of glucose; this trait allows for the calibration curve to be generated, as the data themselves get spaced out over time. Accordingly, the calibration data taken at t = 25 seconds in the graph is shown below:

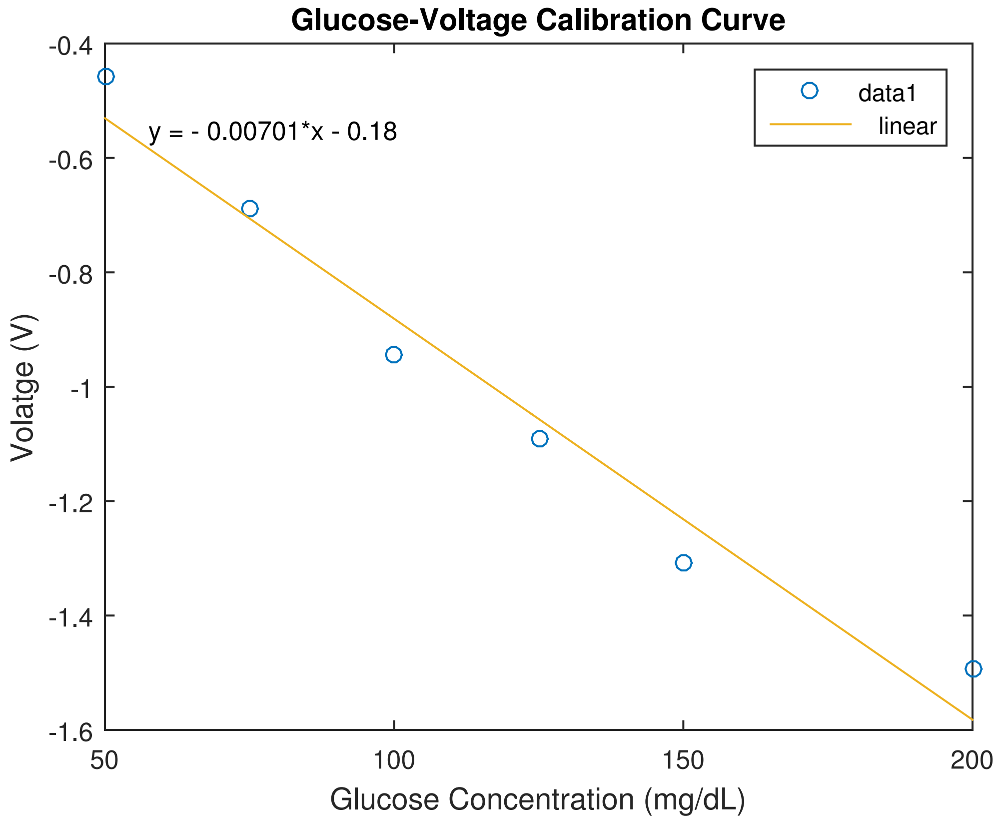


Figure 6: The calibration curve of the Voltage value per Glucose concentration, as taken 25 seconds after the beginning of the interaction.

As seen, the concentration of glucose and the voltage level are linearly dependent. The following figure also depicts the integrated value of the voltage:

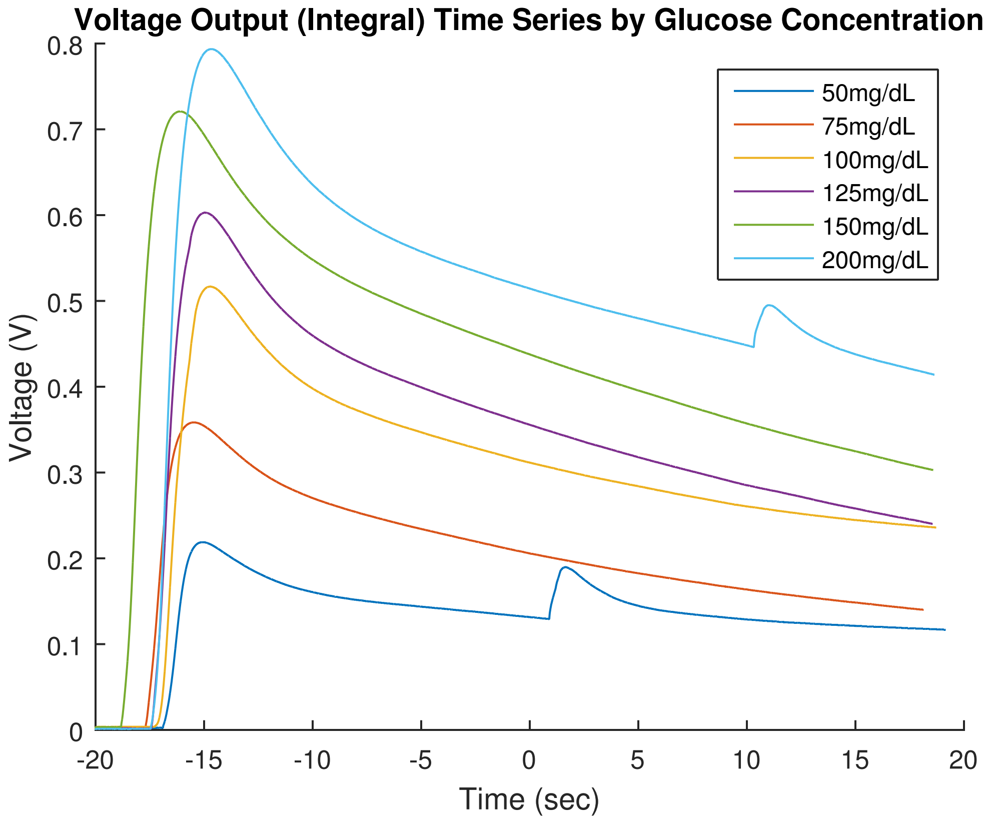


Figure 7: The integrated value of the voltage.

As seen, the fluctuations are much more exposed, thereby rendering it a poor candidate in terms of gauging the calibration.

ISim Lab 7: Blood Pressure

Yoonyoung Cho

October 2015

1 Introduction

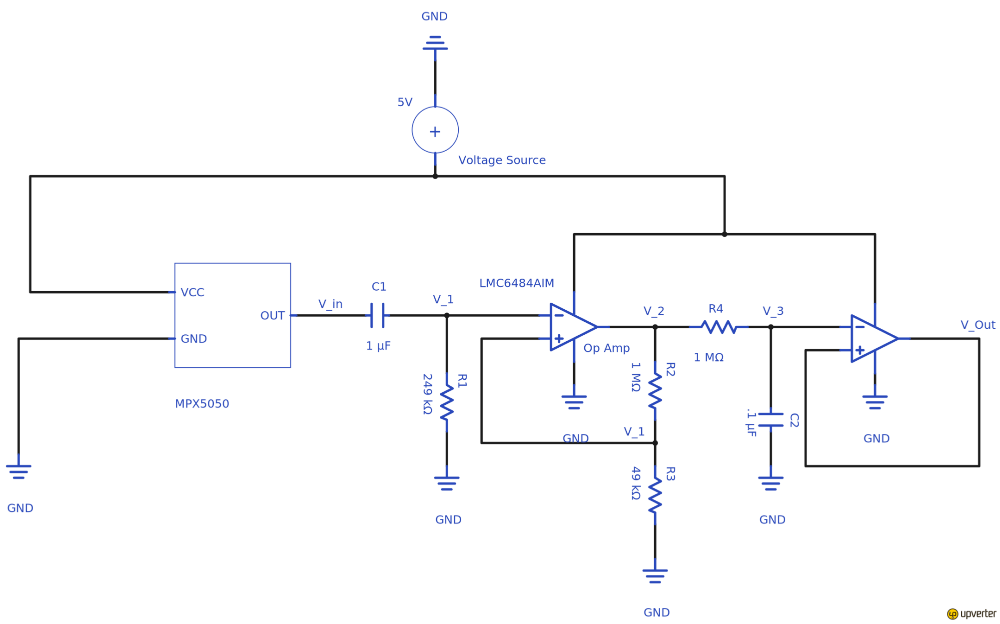


Figure 1: Diagram of the whole circuit, drawn with Upverter.

Since operational Amplifiers draw no current, each subcomponent may be treated as a separate system; naturally, in the following sections, I will process the voltage in each phase separately, rather than combining them into a singular expression that relates V_{out} directly to V_{in}. In this scenario, the circuit is composed of four phases: high-pass filter, amplifier, low-pass filter, and unity-gain follower.[1]

[1] the low-pass filter only depends on the voltage after the amplification, and is more or less a separate system.

2 Circuit Analysis

2.1 High-pass Filter

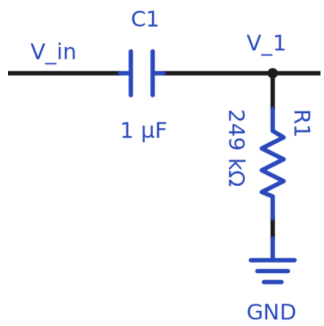


Figure 2: First Phase : High-pass Filter from V_{in} to V_1

$$\frac{V_1 - V_{in}}{1/j\omega C_1} = \frac{0 - V_1}{R_1}$$
$$V_1(j\omega C_1 + 1/R_1) = V_{in} * j\omega C_1$$
$$V_1 = \frac{V_{in} * j\omega C_1}{j\omega C_1 + 1/R_1}$$

2.2 Amplifier

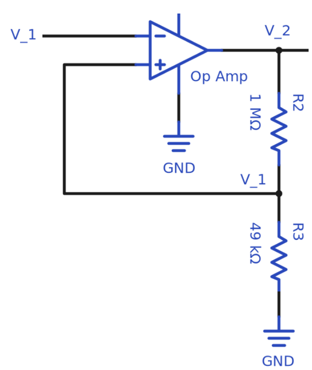


Figure 3: Second Phase : Amplifier from V_1 to V_2

$$\frac{V_1 - 0}{R_3} = \frac{V_2 - V_1}{R_2}$$
$$V_1(1/R_3 + 1/R_2) = \frac{V_2}{R_2}$$
$$V_2 = R_2 V_1 \frac{R_2 + R_3}{R_2 R_3}$$
$$V_2 = V_1 \frac{R_2 + R_3}{R_3}$$

2.3 Low-pass Filter

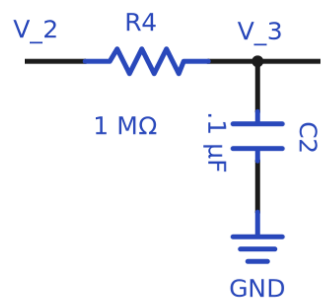


Figure 4: Third Phase : Low-pass Filter from V_2 to V_3

$$\begin{aligned} \frac{V_3 - V_2}{R_4} &= \frac{0 - V_3}{1/j\omega C_2} \\ V_3(1/R_4 + j\omega C_2) &= \frac{V_2}{R_4} \\ V_3 &= \frac{V_2}{1 + R_4 j\omega C_2} \end{aligned}$$

2.4 Unity-Gain Follower

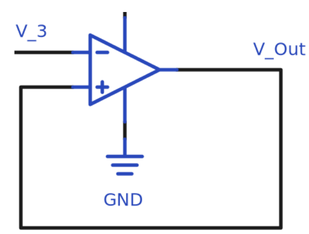


Figure 5: Fourth Phase : Unity-Gain Follower from V_3 to V_{out}

Given that the voltage is not stuck at the extreme values(rails), it is the property of the Op-Amp that $V_{out} == V_3$. This Unity-Gain Follower was implemented in order to facilitate the measurement of voltage, by the Analog Discovery.

3 Results

3.1 Verification

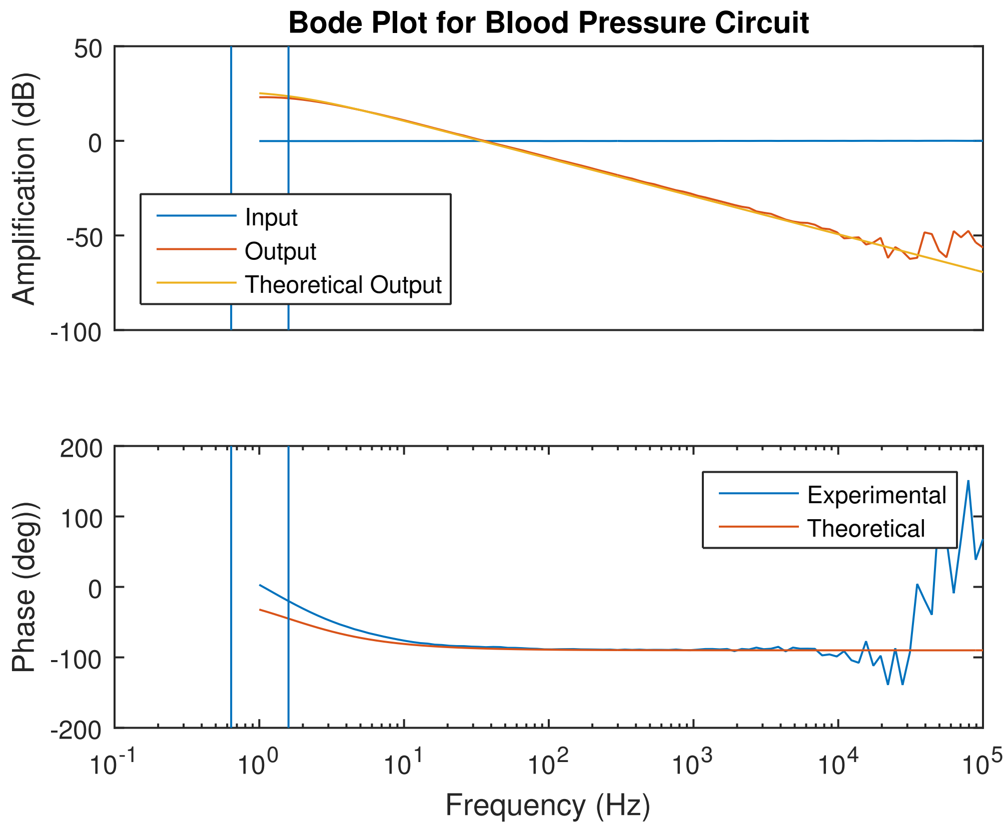


Figure 6: Bode plot of the circuit. The Analog Discovery was unable to create the bode plot below 1 Hz – hence the cutoff in the left part of the plot. Cutoff Frequency for the High-pass filter was .693 Hz, and the cutoff frequency for the Low-pass filter was 1.59 Hz.

Experimental data and Theoretical data showed a clear coherence except at higher frequencies, where the measurement is known to be unstable.

3.2 Application

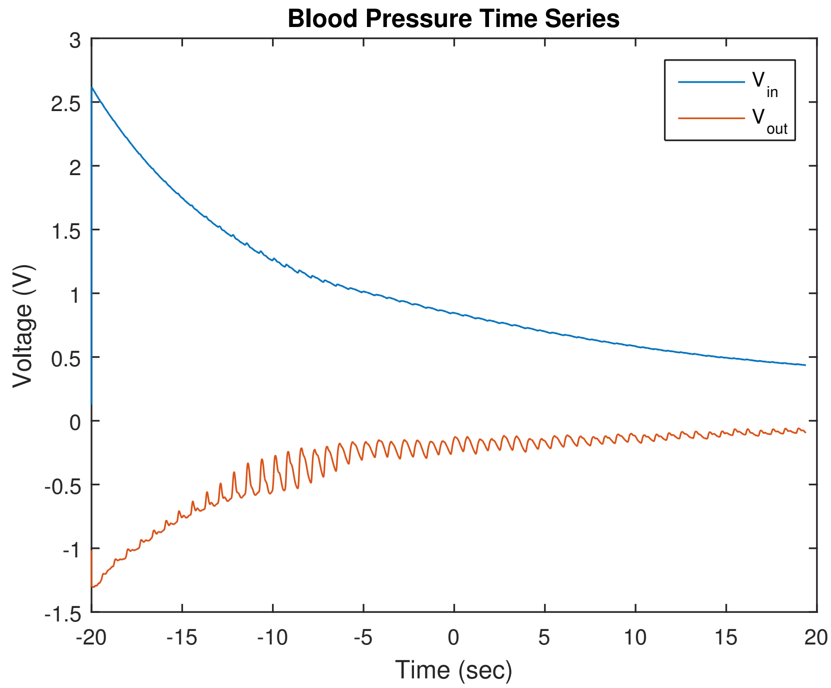


Figure 7: The time series of the Blood Pressure. As shown, the processed signal shows a greater indication of recurring blood pressure than the raw signal.

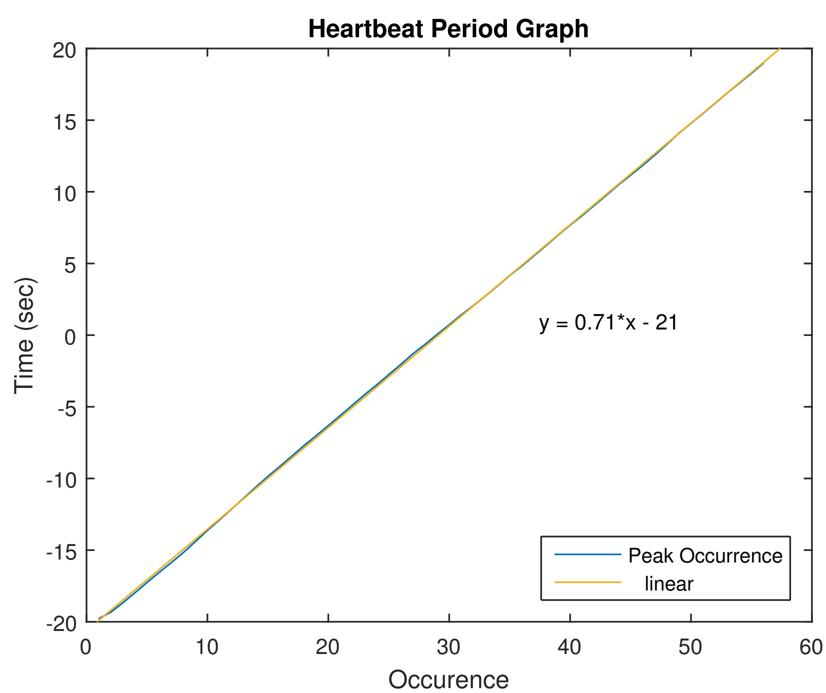


Figure 8: The time series was analyzed to find the period of the blood pressure: as shown in the fit line, each peak occurred every 0.71 seconds.

ISim Lab 8: BCG

Yoonyoung Cho

November 9 2015

1 BCG Trace

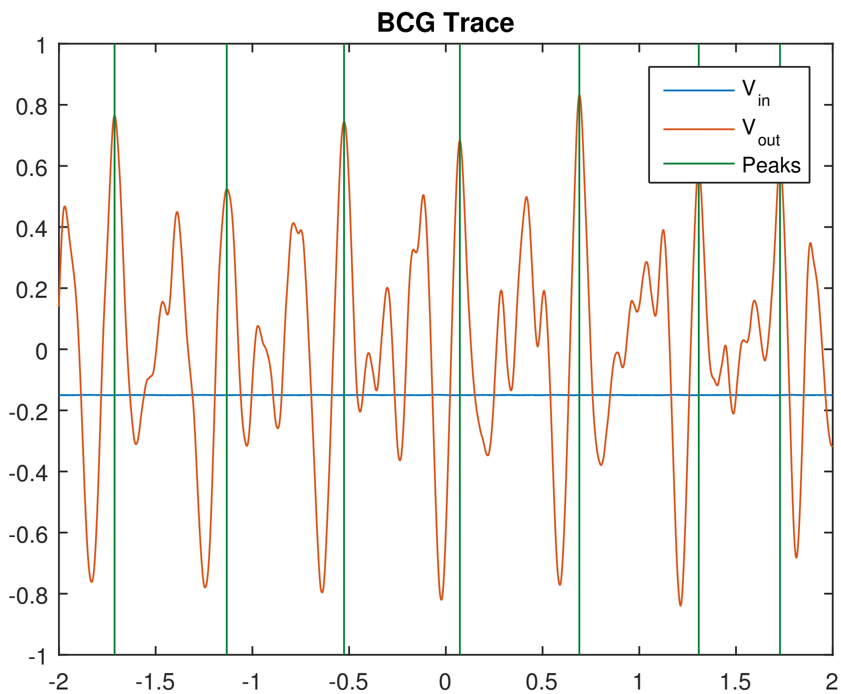


Figure 1: Trace of Ballistocardiogram; a regular pattern of crests preceded by troughs is observed.

2 Period Analysis

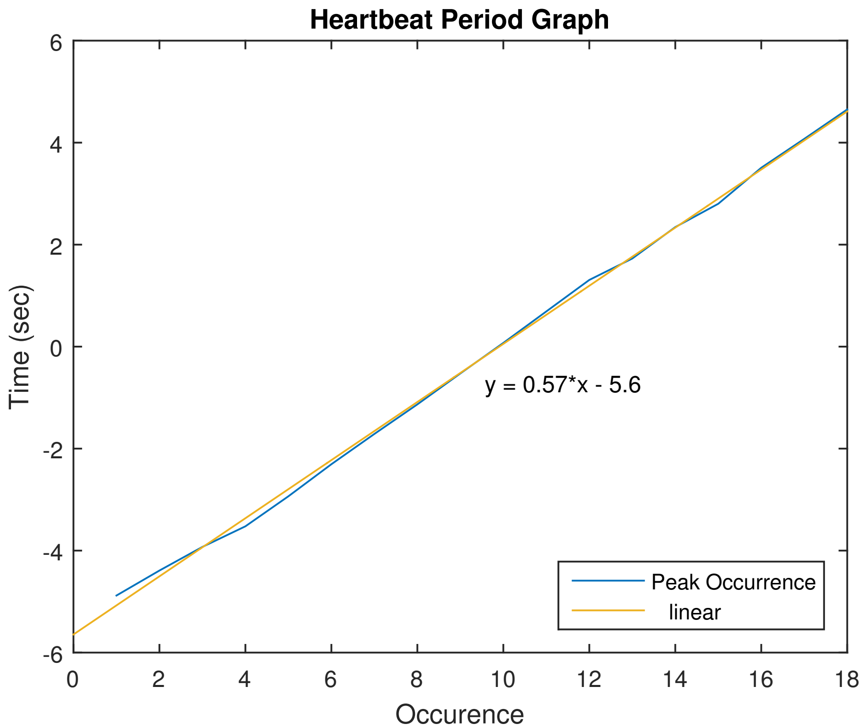


Figure 2: Heartbeat Period; as seen, the peak occurred every .57 seconds.

3 Circuit Diagram

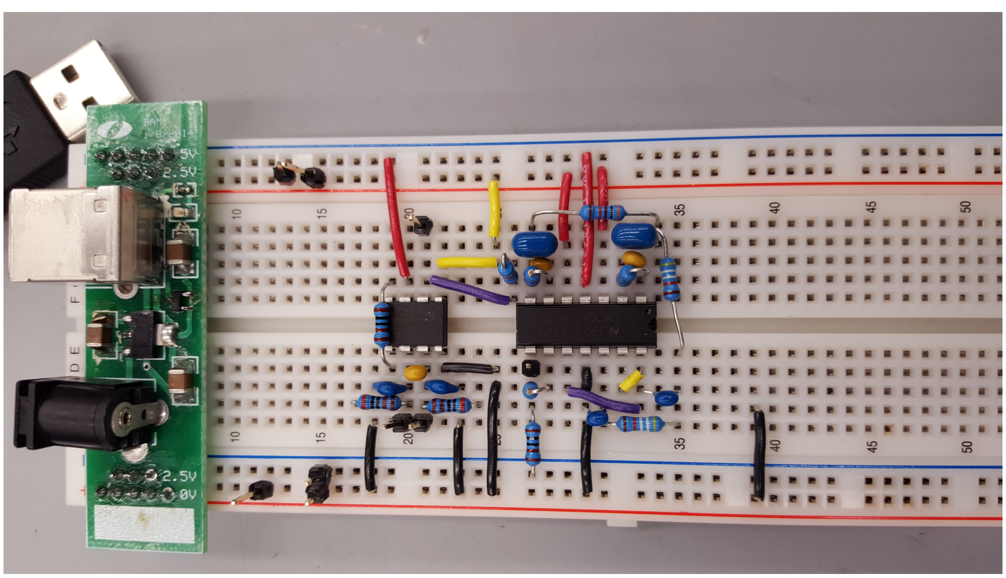


Figure 3: Circuit Diagram.

ISim Lab9

Yoonyoung Cho

November 16 2015

1 Schematic

The full schematic of the receiver circuit is shown below:

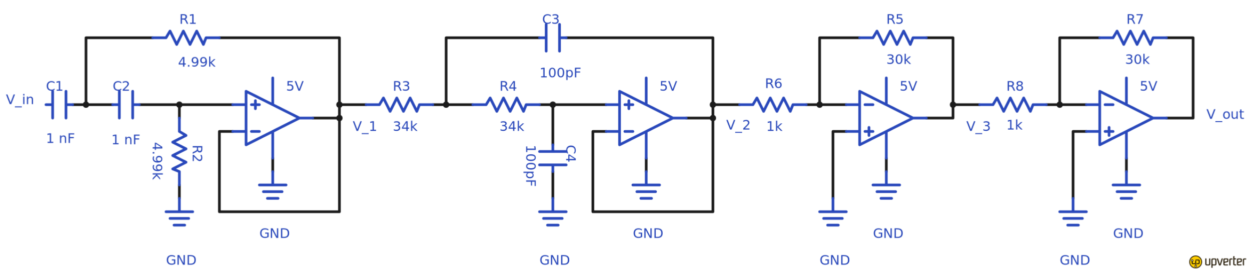


Figure 1: Full schematic of the receiver circuit.

In this circuit, V_{in} corresponds to the raw signal input from the receiver; V_{out} corresponds to the signal after going through a second order band-pass filter composed of two Sallen-Key filters. In order to keep the circuit simple, I did not utilize the amplification capability of the Sallen-Key filters; instead, signal amplification was isolated into a separate process.

2 Analysis

Relying on the fact that Op-Amps draw no current, it makes most sense to break down the parts into subsystems; furthermore, it would be convenient to treat a general case of the Sallen-Key topology, with impedence, that could be applied to both high-pass and low-pass filters. As the two amplifiers in the later stage of the process are identical, the analysis of the overall circuit would just involve two circuit structures.

2.1 Sallen-Key Topology

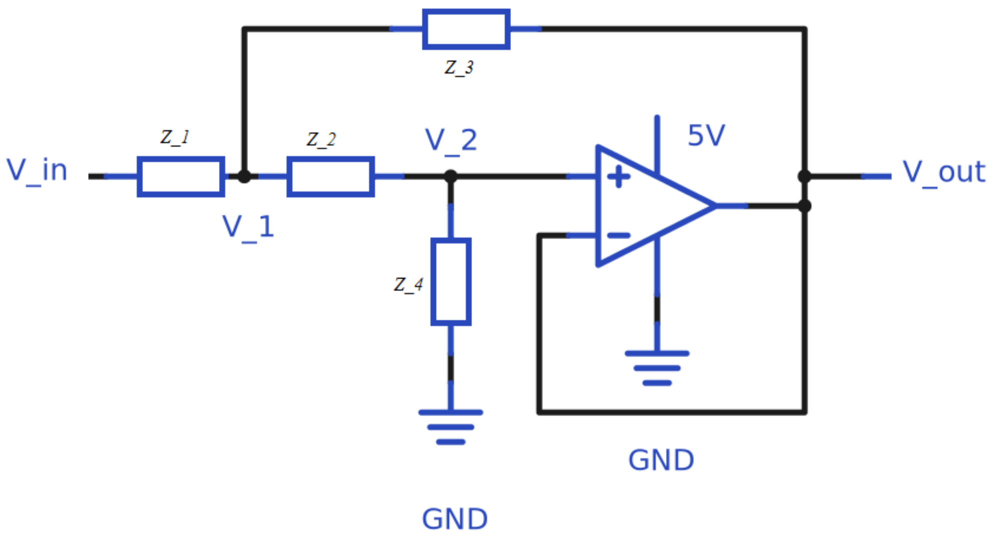


Figure 2: Generic Sallen-Key Topology.

Unlike previous labs, in this circuit the Gain-Bandwidth tradeoff has a significant impact upon; therefore, making a simple assumption that the positive and negative input voltage to the operational amplifiers are inappropriate. Thus, the Op-Amp would be bound to a more complex expression:

$$\frac{dV_{out}}{dt} = \omega_1(V_2 - V_{out})$$

in which ω_1 represents the characteristic frequency of the Op-amp. As well as the usual relationships:

$$\begin{aligned} I &= \frac{V_1 - V_{in}}{Z_1} \\ &= i_1 + i_2 \end{aligned}$$

$$\begin{aligned} i_1 &= \frac{V_{out} - V_1}{Z_3} \\ i_2 &= \frac{V_2 - V_1}{Z_2} \\ &= \frac{0 - V_2}{Z_4} \end{aligned}$$

The above equations conclude the overarching relationships in the circuit. Now, the analysis:

$$\begin{aligned} \frac{V_1}{Z_2} &= \frac{V_2}{Z_2} + \frac{V_2}{Z_4} \\ V_1 &= V_2 \frac{Z_2 + Z_4}{Z_4} \\ V_2 &= V_1 \frac{Z_4}{Z_2 + Z_4} \end{aligned}$$

$$\frac{V_1 - V_{in}}{Z_1} = \frac{V_{out} - V_1}{Z_3} - \frac{V_1}{Z_2 + Z_4}$$
$$V_1(\frac{1}{Z_1} + \frac{1}{Z_3} + \frac{1}{Z_2 + Z_4}) = \frac{V_{in}}{Z_1} + \frac{V_{out}}{Z_3}$$
$$Z_3 V_1 + Z_1 V_1 + \frac{Z_1 Z_3 V_1}{Z_2 + Z_4} = Z_3 V_{in} + Z_1 V_{out}$$
$$V_1(Z_1 + Z_3 + \frac{Z_1 Z_3}{Z_2 + Z_4}) = Z_3 V_{in} + Z_1 V_{out}$$
$$V_1 = \frac{Z_3 V_{in} + Z_1 V_{out}}{Z_1 + Z_3 + \frac{Z_1 Z_3}{Z_2 + Z_4}}$$

$$\frac{dV_{out}}{dt} = \omega_1 (V_2 - V_{out})$$
$$V_2 = V_1 \frac{Z_4}{Z_2 + Z_4}$$
$$\frac{dV_{out}}{dt} = \omega_1 (V_1 \frac{Z_4}{Z_2 + Z_4} - V_{out})$$
$$V_1 = \frac{Z_3 V_{in} + Z_1 V_{out}}{Z_1 + Z_3 + \frac{Z_1 Z_3}{Z_2 + Z_4}}$$
$$\frac{dV_{out}}{dt} = \omega_1 (\frac{Z_3 V_{in} + Z_1 V_{out}}{Z_1 + Z_3 + \frac{Z_1 Z_3}{Z_2 + Z_4}} \frac{Z_4}{Z_2 + Z_4} - V_{out})$$
$$\frac{dV_{out}}{dt} = k_1 V_{in} + k_2 V_{out}$$

Here k_1 and k_2 are simply placeholders for the complex expressions:

$$k_1 = \omega_1 Z_3 (\frac{Z_4}{Z_2 + Z_4} \frac{1}{Z_1 + Z_3 + \frac{Z_1 Z_3}{Z_2 + Z_4}})$$
$$k_2 = \omega_1 (\frac{Z_4}{Z_2 + Z_4} * \frac{1}{Z_1 + Z_3 + \frac{Z_1 Z_3}{Z_2 + Z_4}} Z_1 - 1)$$

$$V_{in} = V_0 e^{j\omega t}$$
$$V_{out} = G V_0 e^{j\omega t}$$
$$j\omega G V_0 e^{j\omega t} = k_1 V_0 e^{j\omega t} + k_2 G V_0 e^{j\omega t}$$
$$j\omega G = k_1 + k_2 G$$
$$G(j\omega - k_2) = k_1$$
$$G = \frac{k_1}{j\omega - k_2}$$

G represents the encoded Gain of V_{out} with respect to V_{in} (as well as the phase shift)

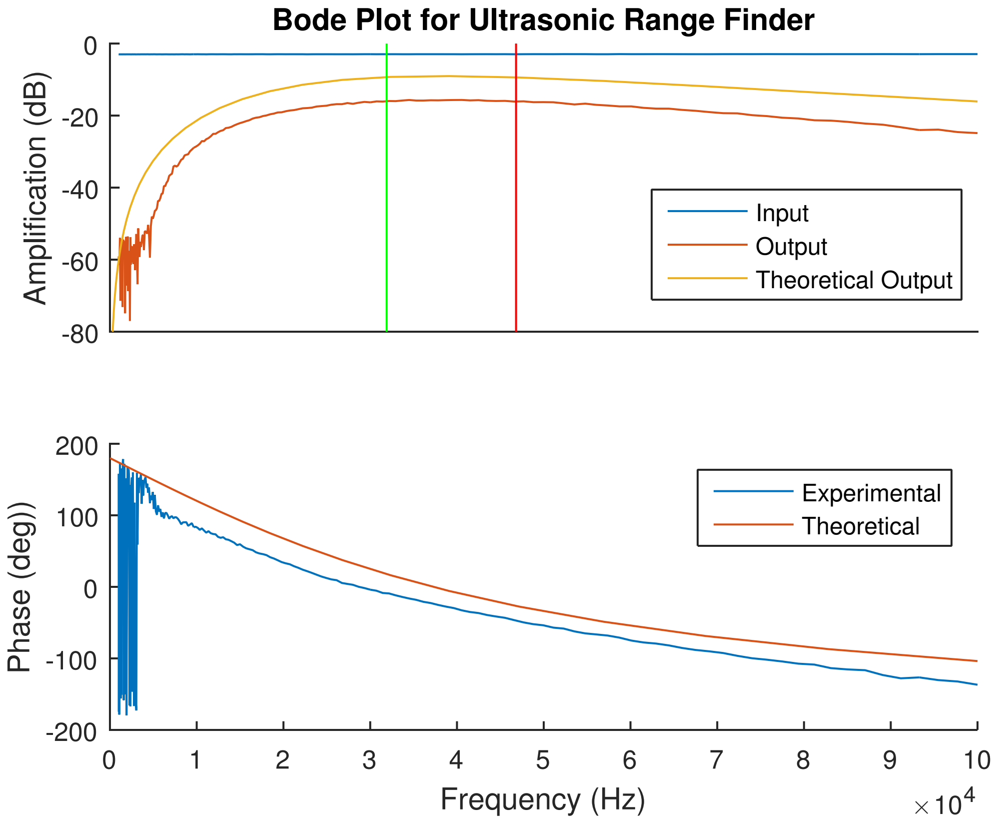


Figure 3: Bode plot, for the Band-pass filter only.

I couldn't get the plot to output reasonable data with the filters and amplifiers in conjunction; the parts were working fine in isolation, so I chose to present the results separately. As shown, the output is consistent with the theoretical output in trends, though with less intensity. As expected, the roll-off is much milder in the higher frequencies, in which the gain-bandwidth tradeoff reduces the overall magnitude, than in the lower frequencies.

2.2 Inverting Amplifier

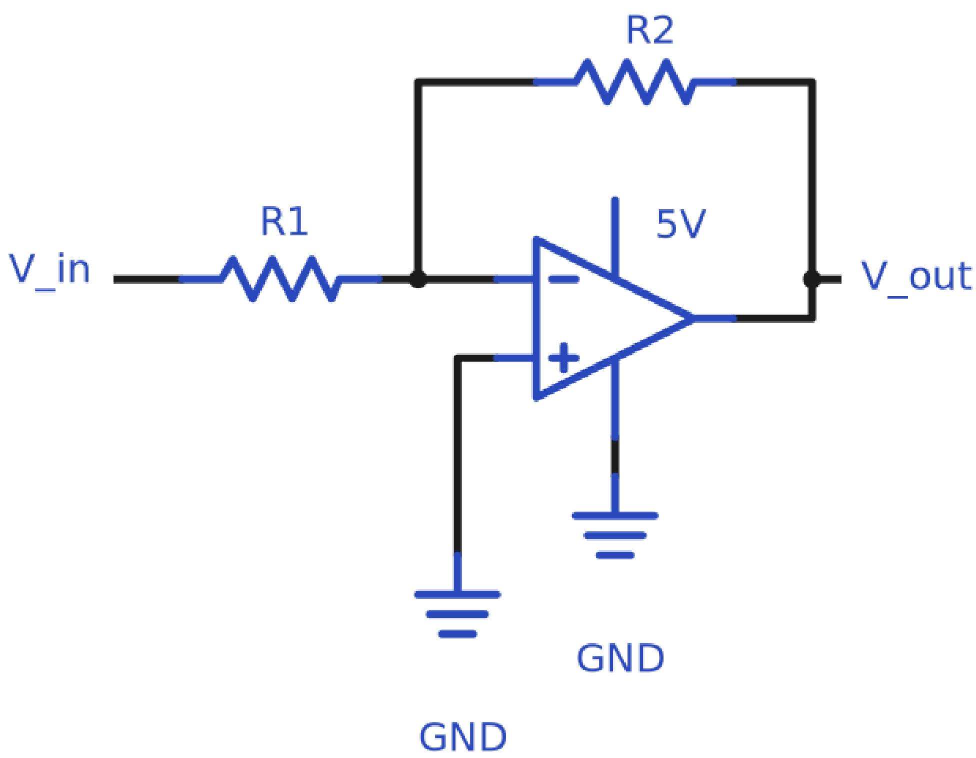


Figure 4: Inverting Amplifier Schematic.

Cutting to the chase, the relationships are as follows:

$$\frac{V_1 - V_{in}}{R_1} = \frac{V_{out} - V_1}{R_2}$$
$$\frac{dV_{out}}{dt} = -\omega_1 V_1$$

Henceforth, the analysis:

$$V_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{V_{out}}{R_2} + \frac{V_{in}}{R_1}$$
$$(R_1 + R_2)V_1 = R_1 V_{out} + R_2 V_{in}$$
$$V_1 = \frac{R_1 V_{out} + R_2 V_{in}}{R_1 + R_2}$$

$$\frac{dV_{out}}{dt} = -\omega_1 \frac{R_1 V_{out} + R_2 V_{in}}{R_1 + R_2}$$
$$\frac{dV_{out}}{dt} = \frac{-\omega_1 R_1}{R_1 + R_2} V_{out} + \frac{-\omega_1 R_2}{R_1 + R_2} V_{in}$$

which is now of the form we had seen before:

$$\frac{dV_{out}}{dt} = k_1 V_{in} + k_2 V_{out}$$
$$k_1 = \frac{-\omega_1 R_2}{R_1 + R_2}$$
$$k_2 = \frac{-\omega_1 R_1}{R_1 + R_2}$$

thus,

$$G = \frac{k_1}{j\omega - k_2}$$

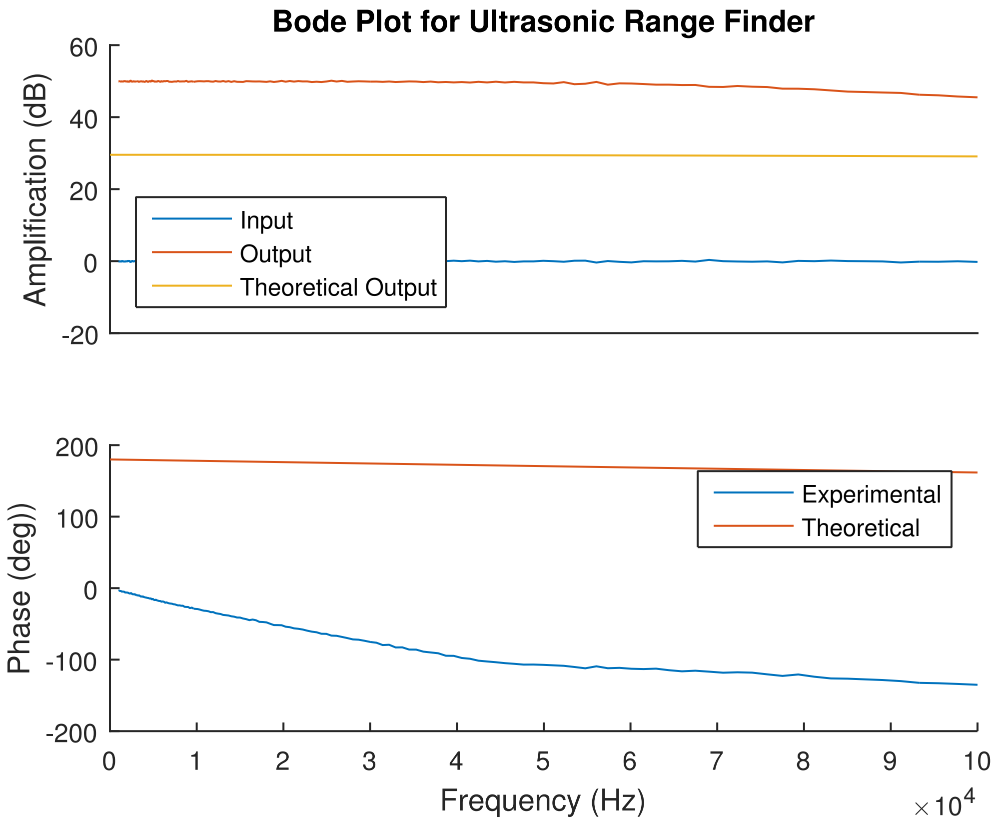
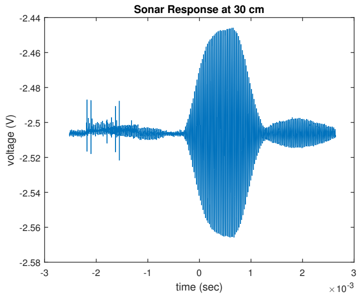


Figure 5: Bode plot, for the Amplifier only.

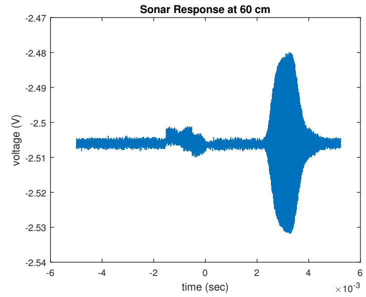
The theory as to the capped amplification of the amplifier was that the voltage emitted by the Analog Discovery was simply too big to be amplified 900 times; the discrepancy for the phase remains unclear, but is fortunately irrelevant to the signal reception in this circuit.

3 Comparison

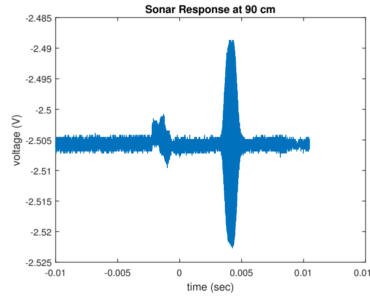
The measured sonar responses are as follows:



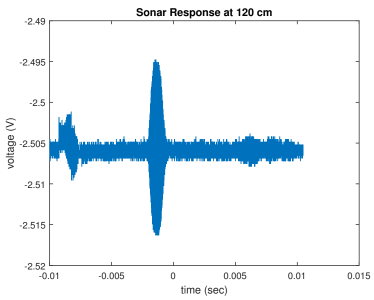
(a) 30 cm



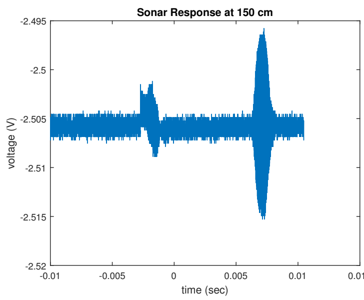
(b) 60 cm



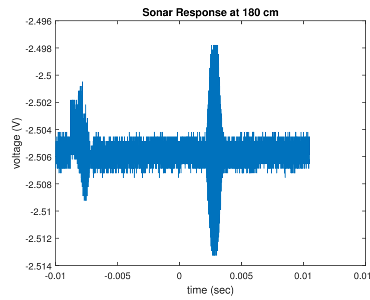
(c) 90 cm



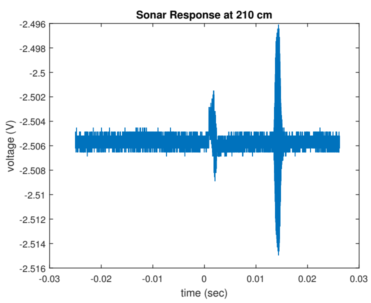
(d) 120 cm



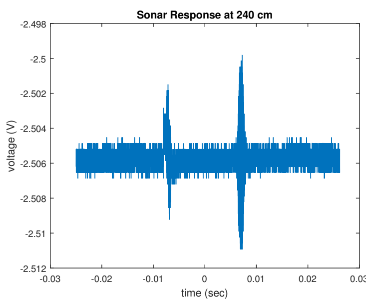
(e) 150 cm



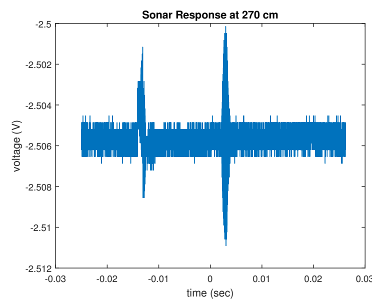
(f) 180 cm



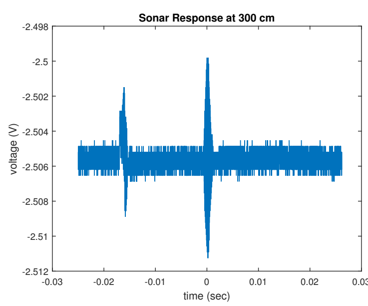
(g) 210 cm



(h) 240 cm



(i) 270 cm



(j) 300 cm

Figure 6: Measured sonar responses.

In order to emphasize the comparison, the data were all combined upon one plot with spaces in between:

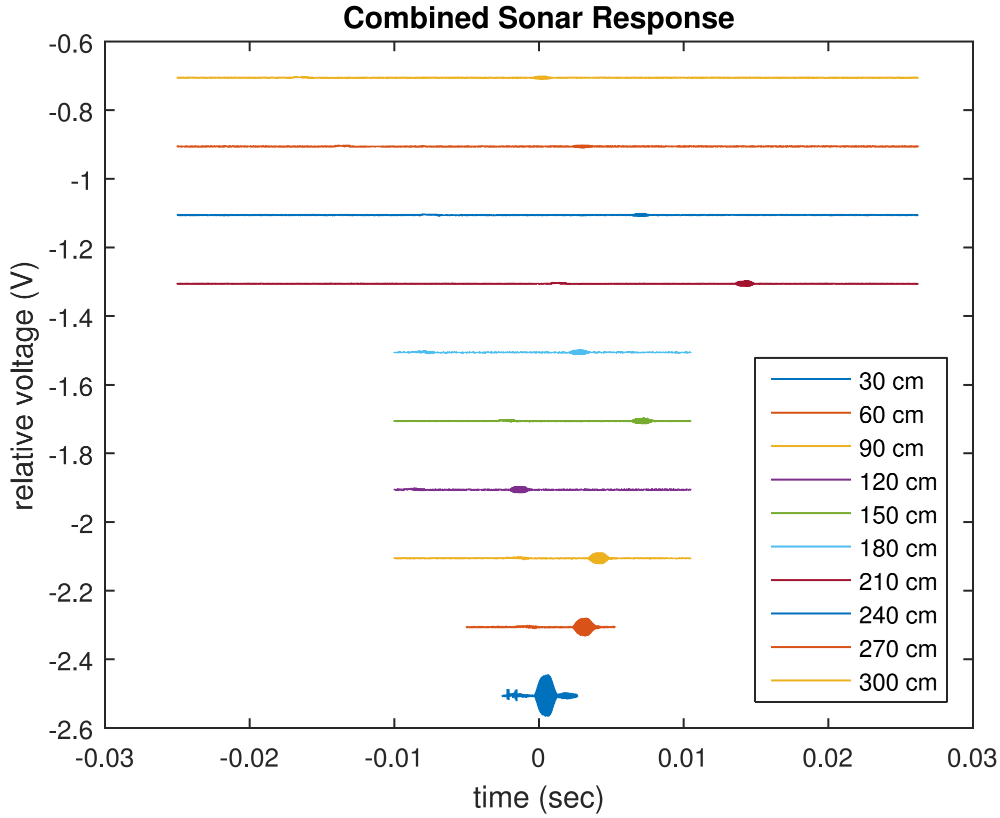


Figure 7: Combined Sonar Response.

The trend becomes more apparent in this display: the magnitude of the signal is significantly greater at closer range, and it takes less time to come back. After digitizing the signals(as it was very difficult to implement an automated process), I obtained the following result:

Table 1: Calculation of distance from the time interval of signals; Val was obtained by digitizing the data.

Dist(cm)	Val	time(sec)	time/2	dist(m)	dist(cm)
30	0.03358	0.002015	0.001007	0.342804	34.2804
60	0.065815	0.003949	0.001974	0.67189	67.18905
90	0.094694	0.005682	0.002841	0.966702	96.6702
120	0.120215	0.007213	0.003606	1.227237	122.7237
150	0.150437	0.009026	0.004513	1.535765	153.5765
180	0.181329	0.01088	0.00544	1.851136	185.1136
210	0.208865	0.012532	0.006266	2.132239	213.2239
240	0.238415	0.014305	0.007152	2.433911	243.3911
270	0.267293	0.016038	0.008019	2.728718	272.8718
300	0.274681	0.016481	0.00824	2.804134	280.4134

From this, I was able to validate my data, which concludes this lab.